Operations on Decimals

Addition and subtraction of decimals

To add decimals, write the numbers so that the decimal points are on a vertical line. Add as you would with whole numbers. Then write the decimal point in the sum directly below the decimal points in the addends.

Add: $0.326 + 4.8 + 57.23$

> **Note that by placing the decimal points on a vertical line, digits of the same place value were added.**

Insert zeros so that it has the same number of decimal places

$$
\begin{array}{r}
0.326 \\
4.800 \\
\underline{57.230} \\
62.356
\end{array}
$$

Subtract and check: 31.642 - 8.759

> Note that by placing the decimal points on a vertical line, digits of the same place value are subtracted.

$$
\begin{array}{r}
31.642 \\
- \quad \underline{8.759} \\
22.883
\end{array}
$$

Check:	**Subtrahend**	8.759
+	**Difference**	+ **22.883**
=	**Minuend**	31.642

The sign rules for adding and subtracting decimals are the same rules used to add and subtract integers.

Recall that the opposite or additive inverse of n is – n and the opposite of –n is n. To find the opposite of a number, change the sign of the number.

Simplify: -2.86 – 10.3

Rewrite subtraction as addition of the opposite.   -2.86 - 10.3

The opposite of 10.3 is – 10.3.   = -2.86 + (-10.3)

The signs of the addends are the same.
Add the absolute values of the numbers.
Attach the sign of the addends.   = -13.16

Recall that to estimate the answer to a calculation, round each number to the highest place value of the number; the first digit of each number will be nonzero and all other digits will be zero. Perform the calculation using the rounded numbers.

Estimate the sum of 23.037 and 16.7892.

Round each number to the nearest ten.   23.037   →→   20
                                                                      16.7892 →→ +20
Add the rounded numbers   40

40 is an estimate of the sum of 23.037 and   23.037
16.7892. Note that 40 is very close to the actual   + 16.7892
 sum of 39.8262.   39.8262

Note: When a number in an estimation is a decimal less than one, round the decimal so that there is one nonzero digit.

Estimate the difference between 4.895 and 0.6193.

Round 4.895 to the nearest one.   4.895   →   5.0
Round 0.6193 to the nearest tenth.   0.6193   →- 0.6
Subtract the rounded numbers.   4.4

4.4 is an estimate of the difference.
It is close to the actual difference of 4.2757.

Multiplication of decimals

Decimals are multiplied as if they were whole numbers; then the decimal point is placed in the product. Writing the decimals as fractions shows where to write the decimal point in the product.

$$0.4 \times 2 = \frac{4}{10} \cdot \frac{2}{1} = \frac{8}{10} = 0.8$$

1 decimal place in 0.4   1 decimal place in 0.8

$$0.4 \cdot 0.2 = \frac{4}{10} \cdot \frac{2}{10} = \frac{8}{100} = 0.08$$

1 decimal place in 0.4   2 decimal places in 0.08
1 decimal place in 0.2

To multiply decimals, multiply the numbers as you would whole numbers. Then write the decimal point in the product so that the number of decimal Places in the product is the sum of the numbers of decimal places in the Factors.

Multiply: (32.41)(7.6)

```
  32.41     2 decimal places
x  7.6      1 decimal place
  19446
 22687
 246.316    3 decimal places
```

Estimating the product of 32.41 and 7.6 shows that the decimal point has been Correctly placed

Round 32.41 to the nearest ten.	32.41	→	30
Round 7.6 to the nearest one.	7.6	→	x 8
Multiply the two numbers.			240

240 is an estimate of (32.41) (7.6). It is close to the actual product 246.316.

To multiply a decimal by a power of 10(10, 100, 1,000,…), move the decimal
point to the right the same number of places as there are zeros in the power
of 10.

$$2.7935 \bullet 1\underline{0} \qquad = 27.935$$

            1 zero   1 decimal place

$$2.7935 \bullet 1\underline{00} \qquad = 279.35$$

           2 zeros   2 decimal places

Note that if the power of 10 is written in exponential notation, the exponent
indicates how many places to move the decimal point.

$$2.7935 \bullet 10^1 \ = 27.935$$

                1 decimal place

$$2.7935 \bullet 10^2 \ = 279.35$$

                2 decimal places

The sign rules for multiplying decimals are the same rules used to multiply integers.

The product of two numbers with the same sign is positive.
The product of two numbers with different signs is negative.

Division of decimals

**To divide decimals, move the decimal point in the divisor to the right so
That the divisor is a whole number. Move the decimal point in the dividend
The same number of places to the right. Place the decimal point in the quotient
directly above the decimal point in the dividend. Then divide as you would with
whole numbers.**

Divide: $29.585 \div 4.85$

$$4.85.\overline{)29.58.5}$$

Move the decimal point 2 places to the right in the divisor.

Move the decimal point 2 places to the right in the dividend.
Place the decimal point in the quotient. Then
divide as shown at the right.

$$\begin{array}{r} 6.1 \\ 485.\overline{)2958.5} \\ -\ 2910 \\ \hline 485 \\ -\ 485 \\ \hline 0 \end{array}$$

Moving the decimal point the same number of places in the divisor and the
Dividend does not change the quotient because the process is the same as
Multiplying the numerator and denominator of a fraction by the same number.

$$4.85\overline{)29.585} = \qquad \frac{29.585}{4.85} = \frac{29.585 \cdot 100}{4.85 \cdot 100} = \frac{2958.5}{485} = 485\overline{)2958.5}$$

To divide a decimal by a power of 10 (10, 100, 1,000, 10,000,…), move the decimal
point to the left the same number of places as there are zeros in the power of 10.

$$462.81 \div 1\underline{0} \quad = 46.281$$

       1 zero   1 decimal place

$$462.81 \div \underline{100} \quad = 4.6281$$

       2 zeros   2 decimal places

$$462.81 \div 1000 = 0.46281$$

       3 zeros   3 decimal places

$$462.81 \div \underline{10,000} \ = \ 0.046281$$
       4 zeros   4 decimal places   A zero must be inserted between the decimal point and the 4.

$$462.81 \div \underline{100,000} \ = \ 0.0046281$$
       5 zeros   5 decimal places   Two zeros must be inserted between the decimal point and the 4.

If the power of 10 is written in exponential notation, the exponent indicates how many places to move the decimal point.

$$462.81 \div 10^1 = 46.281$$

1 decimal place

$$462.81 \div 10^2 = 4.6281$$

2 decimal places

Find the quotient of 3.59 and 100.

There are two zeros in 100. Move the decimal point in 3.59 two places to the left.
$3.59 \div 100 = 0.0359$

What is the quotient of 64.79 and 10^4?

The exponent on 10 is 4. Move the decimal
Point in 64.79 four places to the left. $\qquad$ $64.79 \div 10^4 = 0.006479$

The sign rules for dividing integers are the same rules used to divide decimals.

The quotient of two numbers with the same sign is positive.
The quotient of two numbers with different signs is negative.

Divide: $-1.16 \div 2.9$

The signs are different.
The quotient is negative.
Divide the absolute values of the numbers. $\qquad$ $-1.16 \div 2.9 = -0.4$

Fractions and decimals

Since the fraction bar can be read "divided by," any fraction can be written as a decimal. To write a fraction as a decimal, divide the numerator of the fraction by the denominator.

Convert $\dfrac{3}{4}$ to a decimal.

$$
\begin{array}{r}
0.75 \\
4\,\overline{)3.00} \\
-\underline{28} \\
20 \\
\underline{20} \\
0
\end{array}
$$

← This is a **terminating decimal**.

← The remainder is zero.

To convert a decimal to a fraction, remove the decimal point and place the decimal part over a denominator equal to the place value of the last digit in the decimal.

$$0.57 = \frac{57}{100} \; \lrcorner hundredth\,s \qquad 7.65 = 7\frac{65}{100} = 7\frac{13}{20} \; \lrcorner hundredth\,s$$

$$8.6 = 8\frac{6}{10} = 8\frac{3}{5} \; \lrcorner tenths$$

Convert 4.375 to a fraction.

The 5 in 4.375 is in the thousandths' place.

Write 0.375 as a fraction with a denominator of 1,000.

$$4.375 = 4\frac{375}{1,000}$$

Simplify the fraction.

$$= 4\frac{3}{8}$$