

Introduction to Fractions

A fraction represents equal parts of a whole value.

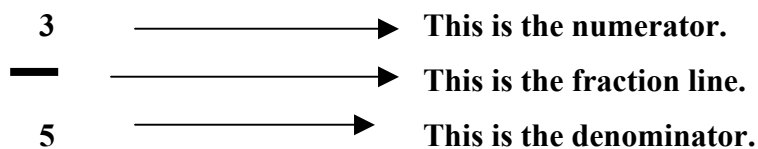
Example:



The shaded area represents one part in four or $\frac{1}{4}$.

The non-shaded area represents three parts of four or $\frac{3}{4}$.

Each part of a fraction has a distinct name:



When the value in the numerator is greater than or equal the denominator, the fraction is known as an *improper* fraction, otherwise it is known as a *proper* fraction.

Proper fractions: $\frac{3}{5}, \frac{1}{4}$

Improper fractions: $\frac{5}{3}, \frac{6}{6}$

Fractions also represent the operation of division.

Therefore, the fraction $\frac{2}{3}$ can be read as “Two divided by three.”

Since fractions are also division operations, the rules applying to division apply also to fractions. These rules include:

$$\frac{0}{a} = 0; a \neq 0$$

$$\frac{a}{a} = 1; a \neq 0$$

$$\frac{a}{0} = \text{undefined}$$

$$\frac{0}{0} = \text{indeterminate}$$

In the case of improper fractions, a value greater than or equal to one is being represented. These fractions may also be written in a form called a mixed number, shown in the example below.

$$\text{Ex. } \frac{5}{2} = 2\frac{1}{2}$$

The examples below demonstrate how to convert between mixed fractions and improper fractions.

Example 1: Improper fraction to a mixed number

$$\frac{7}{2} = 2\frac{3}{2} = 3\frac{1}{2}$$

Example 2: Mixed number to improper fraction

(note: to convert a mixed number into an improper fraction you must first multiply the denominator by the whole number and then add the product to the numerator.)

$$3\frac{5}{8} = \frac{(8 \cdot 3) + 5}{8} = \frac{24 + 5}{8} = \frac{29}{8}$$

Equivalent fractions:

Fractions with different denominators may actually represent the same value.

Examples: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

$$\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$$

When a fraction is in the simplest form, the numerator and denominator have no common factors.

Example 1. Write $\frac{18}{54}$ in its simplest form.

$$\frac{18}{54} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 3 \cdot 3} = \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 3} = \frac{1}{3} \quad \text{Therefore the simplest form of } \frac{18}{54} \text{ is } \frac{1}{3}$$

Example 2. Does $\frac{2}{3} = \frac{8}{12}$?

$$\frac{2}{3} \stackrel{?}{=} \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 3} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot 3} = \frac{2}{3} \quad \text{Therefore } \frac{2}{3} \text{ does equal } \frac{8}{12}$$

Example 3. Write an equivalent fraction.

$$\frac{5}{8} = \frac{x}{40}$$

Step 1. Divide the denominators. $40 \div 8 = 5$

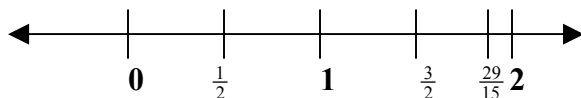
Step 2. Multiply the fraction by the result when placed in a fraction equal to one. $\frac{5}{5} = 1$

$$\left(\frac{5}{8}\right) \cdot \left(\frac{5}{5}\right) = \frac{25}{40}$$

$$\frac{25}{40} = \frac{x}{40}$$

Therefore X = 25

As with all values on the number line, a fraction that is to the left of another is lesser than the one to its right.



Examples. $\frac{1}{2} < \frac{3}{2}$, $\frac{29}{15} > \frac{1}{2}$