Multiplication and Division of Fractions

Multiplication of Fractions

The product of two fractions is the product of the numerators over the product of the denominators.

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$ Where **b ≠ 0** and **d ≠ 0**

Example: Multiply: $\frac{2}{5}\times\frac{1}{3}$

$$\frac{2}{5}\times\frac{1}{3}=\frac{2\times1}{5\times3}=\frac{2}{15}$$ (Multiply the numerators and the denominators)

If a is a natural number, then $1/a$ is called the reciprocal or the multiplicative inverse.

The product of a number and its multiplicative inverse is **1.**

Example: $\frac{1}{8}\times8=8\times\frac{1}{8}=1$

The product:	*Of an **odd number** of negative fractions is **negative***
	*Of an **even number** of negative fractions is **positive***

Example 1: $\frac{-3}{8}\times\frac{-2}{5}\times\frac{-10}{21}$

$\Rightarrow$ There are an **odd number** of negative fractions, so the **product** will be **negative**.

$\Rightarrow$ Use the **Order of Operations Agreement**. Multiply the first two fractions. The **product** is positive.

$$\frac{-3}{8}\times\frac{-2}{5}$$

$\Rightarrow$ The **product** of the first two fractions and the third fraction is **negative**.

$$-\left(\frac{3\times 2\times 10}{8\times 5\times 21}\right)$$

$\Rightarrow$ **Multiply** the **numerators** and the **denominators**.

$$-\left(\frac{60}{840}\right)$$

$\Rightarrow$ Write the product in the **simplest form**.

- First write the prime factorization of each number

$$-\left(\frac{3\times 2\times 2\times 5}{2\times 2\times 2\times 5\times 3\times 7}\right)$$

- Then strikethrough the common factors

$$-\left(\frac{\cancel{3}\times\cancel{2}\times\cancel{2}\times\cancel{5}}{2\times\cancel{2}\times\cancel{2}\times\cancel{5}\times\cancel{3}\times 7}\right)$$

- Write what you have left.

$$-\frac{1}{14}$$

Numerator = 1, because it is a factor of all values.

Denominator = 2×7

Example 2: **Multiply**: $3 \times \frac{5}{8}$

$\Rightarrow$ Write **the whole number** 3 as a fraction $\frac{3}{1}$

$$3 \times \frac{5}{8} = \frac{3}{1} \times \frac{5}{8}$$

$\Rightarrow$ Multiply the fractions.

There are **no common factors** in the numerator and denominator.

$$= \frac{3}{1} \times \frac{5}{8}$$

$\Rightarrow$ Write the improper fraction as a **mixed number**

$$= \frac{15}{8} = 1\frac{7}{8}$$

Example 3: **Is $\frac{-2}{3}$ a solution of the equation $\frac{3}{4}x = \frac{-1}{2}$?**

$\Rightarrow$ Replace "x" with $\frac{-2}{3}$ and then simplify

$$\frac{3}{4} \times \left(\frac{-2}{3} \right) = \frac{-1}{2}$$

$$-\left(\frac{3}{4} \times \frac{2}{3} \right) = \frac{-1}{2}$$

$$-\left(\frac{3 \times 2}{2 \times 2 \times 3} \right) = \frac{-1}{2}$$

$\Rightarrow$ The result is

$$\frac{-1}{2} = \frac{-1}{2}$$

YES, **$\frac{-2}{3}$ is a solution of the equation**.

- **Division of Fractions**
  - *The reciprocal of a fraction* is the fraction with the numerator and denominator interchanged.
  - *Inverting the fractions* is the process of interchanging the numerator and the denominator of a fraction.

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

Example: The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

$\Rightarrow$ To find the **reciprocal** of a **whole** number rewrite the whole number as a **fraction with a denominator of 1**. Then **invert** the fraction.

$$6 = \frac{6}{1}$$

So, the reciprocal of 6 is $\frac{1}{6}$.

Division of Fractions

To divide two fractions, multiply by the reciprocal of the divisor.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$ **Where b ≠ 0, c ≠ 0, and d ≠ 0.**

SIGN RULES FOR DIVIDING POSITVE AND NEGATIVE FRACTIONS

(Same as dividing integers)

The quotient of two numbers with the same sign is positive.

The quotient of two numbers with opposite signs is negative.

Example 1: **Simplify** $\frac{-7}{10} \div \frac{-14}{15}$

$\Rightarrow$ The **signs** are the **same**. The quotient is **positive**.

$$= \frac{7}{10} \div \frac{14}{15}$$

$\Rightarrow$ Rewrite the division as **multiplication by the reciprocal**.

$$= \frac{7 \times 15}{10 \times 14}$$

$\Rightarrow$ **Multiply** and **simplify** the fractions.

$$= \frac{7 \times 3 \times 5}{2 \times 5 \times 2 \times 7} = \frac{3}{4}$$

To divide a fraction and a whole number, first write the whole number as a fraction with a denominator of 1.

Example 2: **Find the quotient of $\frac{2}{3}$ and 4.**

$\Rightarrow$ Write the whole number 4 as the fraction $\frac{4}{1}$.

$$= \frac{2}{3} \div \frac{4}{1}$$

$\Rightarrow$ Rewrite the division as multiplication of the reciprocal.

$$= \frac{2}{3} \times \frac{1}{4}$$

$\Rightarrow$ Multiply the fraction.

$$= \frac{2 \times 1}{3 \times 4}$$

$$= \frac{2 \times 1}{3 \times 2 \times 2} = \frac{1}{6}$$

When a number in a quotient is a mixed number, first write the mixed number as an improper fraction. Then divide the fractions.

Example 3: **Divide** $2 \div 1\frac{1}{4}$**.**

$\Rightarrow$ Write the mixed number $1\frac{1}{4}$ as an improper fraction $\left(\frac{5}{4}\right)$.

$$= \frac{2}{3} \div \frac{5}{4}$$

$\Rightarrow$ Rewrite the division as multiplication by the reciprocal.

$$= \frac{2}{3} \times \frac{4}{5}$$

$\Rightarrow$ Multiply the fractions.

$$= \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$