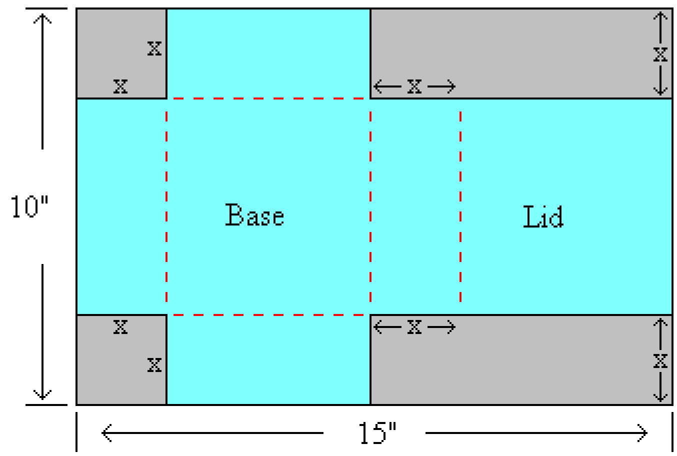
Review Exercise Set 16

Exercise 1: A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a fence. If the farmer only has 2600 feet of fence, what is the largest area that the farmer can enclose? What are the dimensions of the rectangular plot?

Exercise 2: You have been assigned the task of designing a container in the form of a right circular cylinder that can hold a volume of 1000 cubic centimeters. If the cost of the material to be used for the sides costs $0.01 per square centimeter and the cost of the material for the top and bottom costs $0.02 per square centimeter, find the radius, height, and cost of the least expensive container?

Exercise 3: A 10 inch by 15 inch piece of cardboard will be used to construct a box with a lid. Two equal squares will be removed from one side and two equal rectangles are removed from the other corners so that the remaining cardboard can be folded to form the box. Find the value of x that will maximize the volume of the box.

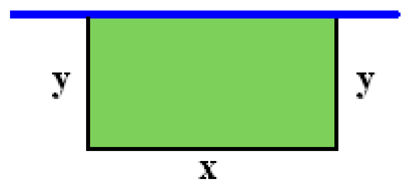


Exercise 4: A retail store purchases computers from a manufacturer. The retail store's current annual demand is 1875 computers. It costs the retailer $3 to store a computer in its inventory and the order costs are $50. What is the optimum number of computers the retailer should purchase in each order and how often must the orders be made?

Review Exercise Set 16 Answer Key

Exercise 1: A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a fence. If the farmer only has 2600 feet of fence, what is the largest area that the farmer can enclose? What are the dimensions of the rectangular plot?

Make drawing of the problem



Define the dimensions in a single variable

We know that the total fencing available is 2600 feet, so the sum of the three sides of fence must equal 2600.

$y + y + x = 2600$
$2y + x = 2600$
$2y = 2600 - x$
$y = 1300 - \frac{1}{2}x$

Setup the equation to be optimized

We need to find the largest area that can be enclosed so we would use the equation for the area of a rectangle.

$A = xy$
$A(x) = x(1300 - \frac{1}{2}x)$

Determine the limitations on function

The sides of the rectangle must be nonnegative so the function is bound by the following inequalities:

$$x \geq 0$$

$$\begin{aligned} 1300 - \frac{1}{2}x &\geq 0 \\ -\frac{1}{2}x &\geq -1300 \\ x &\leq 2600 \end{aligned}$$

So we must find the absolute maximum of A(x) on the closed interval [0, 2600].

Exercise 1 (Continued):

Find the first derivative

$A(x) = x(1300 - \frac{1}{2}x)$

$A(x) = 1300x - \frac{1}{2}x^2$

$A'(x) = 1300 - x$

Find the critical numbers

$0 = 1300 - x$

$x = 1300$

Evaluate the equation at the critical number and interval endpoints

$A(x) = 1300x - \frac{1}{2}x^2$

$A(0) = 1300(0) - \frac{1}{2}(0)^2$

$A(0) = 0$

$A(1300) = 1300(1300) - \frac{1}{2}(1300)^2$

$A(1300) = 1{,}690{,}000 - 845{,}000$

$A(1300) = 845{,}000$

$A(2600) = 1300(2600) - \frac{1}{2}(2600)^2$

$A(2600) = 3{,}380{,}000 - 3{,}380{,}000$

$A(2600) = 0$

The maximum area occurs when $x = 1300$.

Find the value of the other dimension

$y = 1300 - \frac{1}{2}x$

$y = 1300 - \frac{1}{2}(1300)$

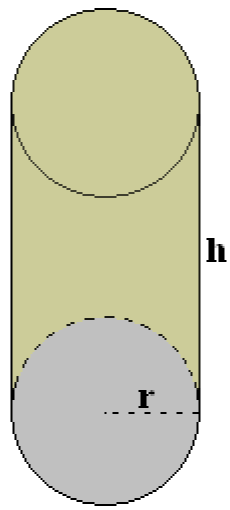
$y = 1300 - 650$

$y = 650$

The dimensions that will maximize the area are 650 feet by 1300 feet and the maximum area will be 845,000 square feet.

Exercise 2: You have been assigned the task of designing a container in the form of a right circular cylinder that can hold a volume of 1000 cubic centimeters. If the cost of the material to be used for the sides costs \$0.01 per square centimeter and the cost of the material for the top and bottom costs \$0.02 per square centimeter, find the radius, height, and cost of the least expensive container?

Make drawing of the problem



Define the dimensions in a single variable

$$
\begin{aligned}
V &= \pi r^2 h \\
1000 &= \pi r^2 h \\
\frac{1000}{\pi r^2} &= h
\end{aligned}
$$

Setup the equation to be optimized

We are looking to minimize the cost of the material used to create the cylinder, so we would need to find the surface area of the cylinder and then multiply by the appropriate costs.

$$
\begin{aligned}
SA &= SA_{top} + SA_{side} + SA_{bottom} \\
&= \pi r^2 + 2\pi rh + \pi r^2 \\
&= 2\pi r^2 + 2\pi rh \\
&= 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right) \\
&= 2\pi r^2 + \frac{2000}{r}
\end{aligned}
$$

Exercise 2 (Continued):

$$C(r) = 2\pi r^2(0.02) + \frac{2000}{r}(0.01)$$
$$= 0.04\pi r^2 + \frac{20}{r}$$

Determine the limitations on function

r > 0

Find the first derivative

$$C(r) = 0.04\pi r^2 + 20r^{-1}$$
$$C'(r) = 0.08\pi r - 20r^{-2}$$
$$= 0.08\pi r - \frac{20}{r^2}$$
$$= \frac{0.08\pi r^3 - 20}{r^2}$$

Find the critical numbers

Set numerator and denominator equal to zero

$$0.08\pi r^3 - 20 = 0$$
$$0.08\pi r^3 = 20$$
$$r^3 = \frac{250}{\pi}$$
$$r = \sqrt[3]{\frac{250}{\pi}}$$
$$r \approx 4.3$$

Find the height when r = 4.3

$$h = \frac{1000}{\pi r^2}$$
$$\approx \frac{1000}{\pi(4.3)^2}$$
$$\approx 17.2$$

Exercise 2 (Continued):

Find the cost when r = 4.3

$C(r) = 0.04\pi r^2 + \frac{20}{r}$

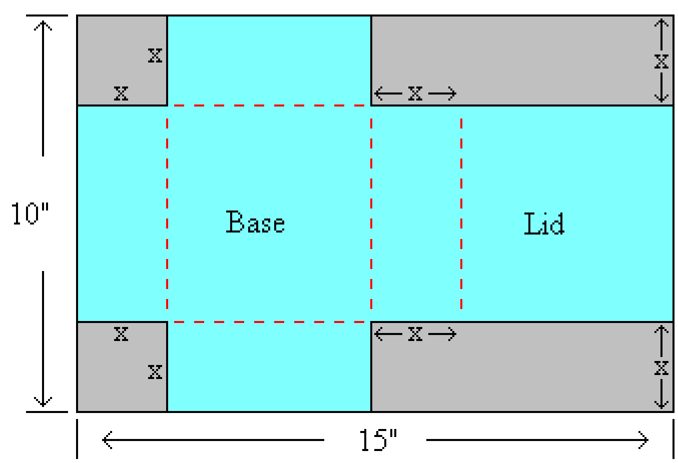
$C(4.3) = 0.04\pi (4.3)^2 + \frac{20}{4.3}$

$C(4.3) \approx 2.32 + 4.65$

$C(4.3) \approx 6.97$

The cylinder would need to have a radius of 4.3 centimeters and a height of 17.2 centimeters in order to minimize the cost at $6.97.

Exercise 3: A 10 inch by 15 inch piece of cardboard will be used to construct a box with a lid. Two equal squares will be removed from one side and two equal rectangles are removed from the other corners so that the remaining cardboard can be folded to form the box. Find the value of x that will maximize the volume of the box.



Define the dimensions in a single variable

We will look at the length first

The length of the base and lid must be the same so for now we will let it be L and there are two lengths of x which is the height of the sides when they are folded up. The total length of the cardboard is 15 inches so:

15 = x + L + x + L

15 = 2x + 2L

15 - 2x = 2L

7.5 - x = L

Exercise 3 (Continued):

Now we will look at the width

The width of the base and lid are not known so we will let them be W for now. The width of the cardboard has two pieces x inches long that will be cut out on the ends. The total width of the cardboard is 10 inches so:

$10 = x + W + x$
$10 = 2x + W$
$10 - 2x = W$

Setup the equation to be optimized

We are looking to maximize the volume of the box so we would use the volume formula for our equation.

$V = LWH$
$V(x) = (7.5 - x)(10 - 2x)(x)$
$V(x) = (75 - 25x + 2x^2)(x)$
$V(x) = 2x^3 - 25x^2 + 75x$

Determine the limitations on function

The measurements cannot be negative so all three are set equal to or greater than zero.

Height	Length	Width
$x \geq 0$	$7.5 - x \geq 0$	$10 - 2x \geq 0$
	$-x \geq -7.5$	$-2x \geq -10$
	$x \leq 7.5$	$x \leq 5$

x is bound by the closed interval [0, 5]

Find the first derivative

$V'(x) = 6x^2 - 50x + 75$

Exercise 3 (Continued):

Find the critical numbers

$0 = 6x^2 - 50x + 75$

$$x = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(6)(75)}}{2(6)}$$

$$= \frac{50 \pm \sqrt{2500 - 1800}}{12}$$

$$= \frac{50 \pm \sqrt{700}}{12}$$

$$x = \frac{50 - \sqrt{700}}{12} \qquad x = \frac{50 + \sqrt{700}}{12}$$

$$x \approx 1.96 \qquad x \approx 6.37$$

x cannot be 6.37 because it is outside of the closed interval [0, 5] so the critical number we would use is 1.96.

Find the second derivative

$V''(x) = 12x - 50$

Test the second derivative at x = 1.96 to confirm it is an absolute maximum

$V''(1.96) = 12(1.96) - 50$
$V''(1.96) = -26.48$

Since the second derivative is negative at x = 1.96 it is concave down and therefore means that this is an absolute maximum.

The volume of the box is maximized when x is 1.96 inches.

Exercise 4: A retail store purchases computers from a manufacturer. The retail store's current annual demand is 1875 computers per year which sell at a uniform rate throughout the year. It costs the retailer \$3 per year to store a computer in its inventory and the order costs are \$50. What is the optimum number of computers the retailer should purchase in each order to minimize the ordering and storage costs? Also, how often are the orders to be made?

Define the variable(s)

x = order size (number of computers per order)

$\frac{1875}{x}$ = number of orders per year

$\frac{x}{2}$ = average inventory (average number of computer stored)

Setup the equation to be optimized

We want to optimize the total ordering and storage costs so our function would be the sum of the ordering costs and storage costs.

C(x) = ordering costs + storage costs

C(x) = (number of orders)(cost/order) + (average inventory)(cost/item)

$C(x) = (\frac{1875}{x})(50) + (\frac{x}{2})(3)$

$C(x) = \frac{93750}{x} + 1.5x$

Find the first derivative

$C(x) = 93750x^{-1} + 1.5x$

$C'(x) = -93750x^{-2} + 1.5$

$C'(x) = -\frac{93750}{x^2} + 1.5$

$C'(x) = \frac{1.5x^2 - 93750}{x^2}$

Find the critical numbers

Set numerator and denominator equal to zero

$1.5x^2 - 93750 = 0$	$x^2 = 0$
$1.5x^2 = 93750$	$x = 0$
$x^2 = 62500$	
$x = \pm 250$	

x cannot be zero or negative, so the only critical number is 250.

Exercise 4 (Continued):

Find the second derivative

$C'(x) = -93750x^{-2} + 1.5$
$C''(x) = 187500x^{-3}$
$C''(x) = \frac{187500}{x^3}$

Test the second derivative at x = 250 to confirm it is an absolute minimum

$C''(250) = \frac{187500}{(250)^3}$
$C''(250) = 0.012$

Since $C''(250)$ is greater than zero, this would be the location of the absolute minimum for the cost function.

Find how many orders need to be made

$\frac{1875}{250}$ = number of orders
7.5 = number of orders