

Review Exercise Set 7

Exercise 1: Find the derivative of the given function.

$$f(u) = (12u^2 + 5)^6$$

Exercise 2: Find the derivative of the given function.

$$V(x) = (2x^2 + 5)^{2/3}$$

Exercise 3: Find the derivative of the given function.

$$f(x) = (2x^3 + 5x)^2 (x^2 - 7)^{2/3}$$

Exercise 4: Find the derivative of the given function.

$$f(x) = (x^2 + 2)^3 / (3x - 5)^2$$

Exercise 5: Find the equation of the tangent line to the function $f(x)$ at the given value of x .

$$f(x) = 2x(5x + 9)^{1/2} + 12 ; x = 8$$

Review Exercise Set 7 Answer Key

Exercise 1: Find the derivative of the given function.

$$f(u) = (12u^2 + 5)^6$$

$$f'(u) = 6 * (12u^2 + 5)^5 * D_u(12u^2 + 5)$$

$$f'(u) = 6(12u^2 + 5)^5(24u)$$

$$f'(u) = 144u(12u^2 + 5)^5$$

Exercise 2: Find the derivative of the given function.

$$V(x) = (2x^2 + 5)^{2/3}$$

$$V'(x) = \frac{2}{3} * (2x^2 + 5)^{-1/3} * D_x(2x^2 + 5)$$

$$V'(x) = \frac{2}{3} (2x^2 + 5)^{-1/3}(4x)$$

$$V'(x) = \frac{8x}{3} (2x^2 + 5)^{-1/3}$$

Exercise 3: Find the derivative of the given function.

$$f(x) = (2x^3 + 5x)^2 (x^2 - 7)^{2/3}$$

Find the derivatives of each term

$$\begin{aligned} D_x(2x^3 + 5x)^2 &= 2(2x^3 + 5x)D_x(2x^3 + 5x) \\ &= 2(2x^3 + 5x)(6x^2 + 5) \\ &= (2x^3 + 5x)(12x^2 + 10) \end{aligned}$$

$$\begin{aligned} D_x(x^2 - 7)^{2/3} &= \frac{2}{3} (x^2 - 7)^{-1/3} D_x(x^2 - 7) \\ &= \frac{4x}{3} (x^2 - 7)^{-1/3} \end{aligned}$$

Exercise 3 (Continued):

Apply the product rule

$$f'(x) = (2x^3 + 5x)^2 D_x(x^2 - 7)^{2/3} + (x^2 - 7)^{2/3} D_x(2x^3 + 5x)^2$$

$$f'(x) = (2x^3 + 5x)^2 \left(\frac{4x}{3}\right)(x^2 - 7)^{-1/3} + (x^2 - 7)^{2/3}(2x^3 + 5x)(12x^2 + 10)$$

$$f'(x) = (2x^3 + 5x)(x^2 - 7)^{-1/3} \left[(2x^3 + 5x) \left(\frac{4x}{3}\right) + (x^2 - 7)(12x^2 + 10) \right]$$

$$f'(x) = (2x^3 + 5x)(x^2 - 7)^{-1/3} \left[\frac{8}{3}x^4 + \frac{20}{3}x^2 + 12x^4 + 10x^2 - 84x^2 - 70 \right]$$

$$f'(x) = (2x^3 + 5x)(x^2 - 7)^{-1/3} \left(\frac{44}{3}x^4 - \frac{202}{3}x^2 - 70 \right)$$

Exercise 4: Find the derivative of the given function.

$$f(x) = (x^2 + 2)^3 / (3x - 5)^2$$

Find the derivatives of each term

$$\begin{aligned} D_x(x^2 + 2)^3 &= 3(x^2 + 2)^2 D_x(x^2 + 2) \\ &= 3(x^2 + 2)^2(2x) \\ &= 6x(x^2 + 2)^2 \end{aligned}$$

$$\begin{aligned} D_x(3x - 5)^2 &= 2(3x - 5) D_x(3x - 5) \\ &= 2(3x - 5)(3) \\ &= 6(3x - 5) \end{aligned}$$

Apply the quotient rule

$$\begin{aligned} f'(x) &= \frac{(3x-5)^2 D_x(x^2+2)^3 - (x^2+2)^3 D_x(3x-5)^2}{\left[(3x-5)^2\right]^2} \\ &= \frac{(3x-5)^2(6x)(x^2+2)^2 - (x^2+2)^3(6)(3x-5)}{(3x-5)^4} \\ &= \frac{(6)(3x-5)(x^2+2)^2 \left[(3x-5)(x) - (x^2+2) \right]}{(3x-5)^4} \\ &= \frac{6(x^2+2)^2(3x^2-5x-x^2-2)}{(3x-5)^3} \\ &= \frac{6(x^2+2)^2(2x^2-5x-2)}{(3x-5)^3} \end{aligned}$$

Exercise 5: Find the equation of the tangent line to the function $f(x)$ at the given value of x .

$$f(x) = 2x(5x + 9)^{1/2} + 12; x = 8$$

Find the derivative

$$f'(x) = (2x) D_x(5x + 9)^{1/2} + (5x + 9)^{1/2} D_x(2x) + 0$$

$$f'(x) = (2x) \frac{1}{2} (5x + 9)^{-1/2}(5) + (5x + 9)^{1/2}(2)$$

$$f'(x) = 5x(5x + 9)^{-1/2} + 2(5x + 9)^{1/2}$$

$$f'(x) = (5x + 9)^{-1/2}[5x + 2(5x + 9)]$$

$$f'(x) = (5x + 9)^{-1/2}(5x + 10x + 18)$$

$$f'(x) = (5x + 9)^{-1/2}(15x + 18)$$

Evaluate the derivative at $x = 8$ to find the solve of the tangent line

$$f'(8) = [5(8) + 9]^{-1/2}[15(8) + 18]$$

$$f'(8) = (49)^{-1/2}(138)$$

$$f'(8) = \frac{138}{\sqrt{49}}$$

$$f'(8) = \frac{138}{7}$$

Evaluate the original function at $x = 8$ to find a point on the tangent line

$$f(8) = 2(8)[5(8) + 9]^{1/2} + 12$$

$$f(8) = (16)(49)^{1/2} + 12$$

$$f(8) = (16)(7) + 12$$

$$f(8) = 124$$

Substitute the slope $\frac{138}{7}$ and point $(8, 124)$ into the point-slope equation of a line

$$y - y_1 = m(x - x_1)$$

$$y - 124 = \frac{138}{7} (x - 8)$$

$$y = \frac{138}{7} x - \frac{1104}{7} + 124$$

$$y = \frac{138}{7} x - \frac{236}{7}$$