Review Exercise Set 18

Exercise 1: Find the derivative of the given logarithmic function.

$f(x) = x \ln (x^3 - 3x)^2$

Exercise 2: Find the derivative of the given logarithmic function.

$y = (2x^2 - 5x + 3) / \ln (x^3 - 1)$

Exercise 3: Find the derivative of the given logarithmic function.

$s(t) = \ln [(2t + 1)^3(5t^2 - t)]$

Exercise 4: Find the derivative of the given logarithmic function.

$$y = \left(e^{2u-3} + 1\right)\left(\ln\sqrt{3u+4}\right)^3$$

Exercise 5: Find the number of units that need to be manufactured so that the profit is maximized given the following revenue and cost functions.

$R(x) = \ln (3x + 1)^{25}$; $C(x) = 5x + 13$

Review Exercise Set 18 Answer Key

Exercise 1: Find the derivative of the given logarithmic function.

$f(x) = x \ln (x^3 - 3x)^2$

Apply the logarithmic properties to simplify the expression

$f(x) = 2x \ln (x^3 - 3x)$

Find derivative of each term

$D_x(2x) = 2$

$D_x \ln (x^3 - 3x) = \frac{D_x(x^3 - 3x)}{x^3 - 3x}$

$D_x \ln (x^3 - 3x) = \frac{3x^2 - 3}{x^3 - 3x}$

Substitute derivatives into the product rule

$f'(x) = 2x * D_x \ln (x^3 - 3x) + \ln (x^3 - 3x) * D_x(2x)$

$f'(x) = 2x * \frac{3x^2 - 3}{x^3 - 3x} + \ln (x^3 - 3x)$

$f'(x) = 2x * \frac{3x^2 - 3}{x(x^2 - 3)} + 2 \ln (x^3 - 3x)$

$f'(x) = \frac{6x^2 - 6}{x^2 - 3} + \ln (x^3 - 3x)^2$

Exercise 2: Find the derivative of the given logarithmic function.

$y = (2x^2 - 5x + 3) / \ln (x^3 - 1)$

Find derivative of each term

$D_x(2x^2 - 5x + 3) = 4x - 5$

$D_x(\ln (x^3 - 1)) = \frac{D_x(x^3 - 1)}{x^3 - 1}$

$D_x(\ln (x^3 - 1)) = \frac{3x^2}{x^3 - 1}$

Exercise 2 (Continued):

Substitute derivatives into the quotient rule

$$y' = \frac{\ln\left(x^3-1\right)D_x\left(2x^2-5x+3\right)-\left(2x^2-5x+3\right)D_x\ln\left(x^3-1\right)}{\left[\ln\left(x^3-1\right)\right]^2}$$

$$= \frac{\left[\ln\left(x^3-1\right)\right](4x-5)-\left(2x^2-5x+3\right)\frac{3x^2}{x^3-1}}{\left[\ln\left(x^3-1\right)\right]^2}$$

$$= \frac{\left[\ln\left(x^3-1\right)\right](4x-5)-\left(2x^2-5x+3\right)\frac{3x^2}{x^3-1}}{\left[\ln\left(x^3-1\right)\right]^2} \times \frac{x^3-1}{x^3-1}$$

$$= \frac{(4x-5)\left(x^3-1\right)\ln\left(x^3-1\right)-\left(2x^2-5x+3\right)\left(3x^2\right)}{\left(x^3-1\right)\left[\ln\left(x^3-1\right)\right]^2}$$

Exercise 3: Find the derivative of the given logarithmic function.

$s(t) = \ln\left[(2t + 1)^3(5t^2 - t)\right]$

Apply the logarithmic properties to simplify the equation

$s(t) = \ln(2t + 1)^3 + \ln(5t^2 - t)$
$s(t) = 3\ln(2t + 1) + \ln(t)(5t - 1)$
$s(t) = 3\ln(2t + 1) + \ln(t) + \ln(5t - 1)$

Take the derivative of each logarithmic term

$$s'(t) = 3\times\frac{D_t(2t+1)}{2t+1}+\frac{D_t(t)}{t}+\frac{D_t(5t-1)}{5t-1}$$

$$= 3\times\frac{2}{2t+1}+\frac{1}{t}+\frac{5}{5t-1}$$

$$= \frac{6}{2t+1}+\frac{1}{t}+\frac{5}{5t-1}$$

Exercise 4: Find the derivative of the given logarithmic function.

$$y = \left(e^{2u-3} + 1\right)\left(\ln\sqrt{3u+4}\right)^3$$

Find the derivative of each term

$D_u\,(e^{2u-3} + 1) = e^{2u-3} * D_u\,(2u - 3) + 0$
$D_u\,(e^{2u-3} + 1) = e^{2u-3} * (2)$
$D_u\,(e^{2u-3} + 1) = 2e^{2u-3}$

$$D_u\left(\ln\sqrt{3u+4}\right)^3 = 3\left(\ln\sqrt{3u+4}\right)^2 D_u\left(\ln\sqrt{3u+4}\right)$$

$$= 3\left(\ln\sqrt{3u+4}\right)^2 \frac{D_u\left(3u+4\right)^{1/2}}{\left(3u+4\right)^{1/2}}$$

$$= 3\left(\ln\sqrt{3u+4}\right)^2 \frac{\frac{1}{2}\left(3u+4\right)^{-1/2}(3)}{\left(3u+4\right)^{1/2}}$$

$$= 3\left(\ln\sqrt{3u+4}\right)^2 \frac{\frac{3}{2}}{\left(3u+4\right)}$$

$$= \left(\ln\sqrt{3u+4}\right)^2 \frac{9}{2\left(3u+4\right)}$$

Apply the product rule to the function

$$y' = \left(e^{2u-3} + 1\right) D_u\left(\ln\sqrt{3u+4}\right)^3 + \left(\ln\sqrt{3u+4}\right)^3 D_u\left(e^{2u-3} + 1\right)$$

Substitute in the derivatives of the terms

$$y' = \left(e^{2u-3} + 1\right)\left(\ln\sqrt{3u+4}\right)^2 \frac{9}{2\left(3u+4\right)} + \left(\ln\sqrt{3u+4}\right)^3\left(2e^{2u-3}\right)$$

Exercise 5: Find the number of units that need to be manufactured so that the profit is maximized given the following revenue and cost functions.

$R(x) = \ln (3x + 1)^{25}$; $C(x) = 5x + 13$

Find the profit function

$P(x) = R(x) - C(x)$
$P(x) = \ln (3x + 1)^{25} - (5x + 13)$
$P(x) = 25 \ln (3x + 1) - 5x - 13$

Find the marginal profit function

$$P'(x) = 25 * \frac{D_x(3x+1)}{3x+1} - 5 - 0$$
$$P'(x) = 25 * \frac{3}{3x+1} - 5$$
$$P'(x) = \frac{75}{3x+1} - 5$$

Set marginal profit function equal to zero and solve for x

$$0 = \frac{75}{3x+1} - 5$$
$$5 = \frac{75}{3x+1}$$
$5(3x + 1) = 75$
$15x + 5 = 75$
$15x = 70$
$x = 4.67$

Find second derivative

$P'(x) = 75(3x + 1)^{-1} - 5$
$P''(x) = -75(3x + 1)^{-2}(3) - 0$
$$P''(x) = -\frac{225}{(3x+1)^2}$$

Exercise 5 (Continued):

Evaluate second derivative with critical number to test for absolute maximum

$$P''(4.67) = -\frac{225}{\left(3\left(4.67\right)+1\right)^2}$$

$P''(4.67) \approx -1$

Since the second derivative is concave down 4.67 is the location that will maximize the profit function

Determine number of units to produce

The critical number is a decimal but the company cannot produce a fractional unit so we would evaluate the profit function at 4 and 5 units to determine which will maximize the profit.

P(4) = 25 ln (3(4) + 1) - 5(4) - 13
P(4) = 25 ln (13) - 20 - 13
P(4) = 25 ln (13) - 33
P(4) $\approx$ 31.12

P(5) = 25 ln (3(5) + 1) - 5(5) - 13
P(5) = 25 ln (16) - 25 - 13
P(5) = 25 ln (16) - 38
P(5) $\approx$ 31.31

The number of units to be produce to maximize the profit function is 5 units.