Review Exercise Set 4

Exercise 1: Find the average rate of change for the function over the given interval.

$f(x) = x^2 + x - 12$ over the interval [-2, 4]

Exercise 2: Find the instantaneous rate of change for the function at the given value.

$f(x) = \frac{3x}{x-4}$ at $x = 2$

Exercise 3: Using the definition for the derivative, find $f'(x)$.

$f(x) = 2x^2 + 3x$

Exercise 4: Using the definition for the derivative, find f´(x) and then evaluate it at x = -3.

$$f(x) = \frac{2}{x+4}$$

Review Exercise Set 4 Answer Key

Exercise 1: Find the average rate of change for the function over the given interval.

$f(x) = x^2 + x - 12$ over the interval [-2, 4]

Average rate of change = $\dfrac{f(x+h)-f(x)}{h}$

Find h.

h is the difference between the ending and starting interval values

h = 4 - (-2)
h = 6

Find the average rate of change

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{f(4)-f(-2)}{6}\\ &= \frac{(4^2+4-12)-\left[(-2)^2+(-2)-12\right]}{6}\\ &= \frac{8-(-10)}{6}\\ &= 3\end{aligned}$$

Exercise 2: Find the instantaneous rate of change for the function at the given value.

f(x) = $\dfrac{3x}{x-4}$ at x = 2

Instantaneous rate of change = $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Find f(x + h)

$$\begin{aligned}f(x+h) &= \frac{3(x+h)}{(x+h)-4}\\ &= \frac{3x+3h}{x+h-4}\end{aligned}$$

Exercise 2 (Continued):

Find f(x + h) - f(x)

$$
\begin{aligned}
f(x+h)-f(x) &= \frac{3x+3h}{x+h-4}-\frac{3x}{x-4} \\
&= \frac{(3x+3h)(x-4)-3x(x+h-4)}{(x+h-4)(x-4)} \\
&= \frac{3x^2-12x+3hx-12h-3x^2-3hx+12x}{(x+h-4)(x-4)} \\
&= \frac{\cancel{3x^2}-\cancel{12x}+\cancel{3hx}-12h-\cancel{3x^2}-\cancel{3hx}+\cancel{12x}}{(x+h-4)(x-4)} \\
&= \frac{-12h}{(x+h-4)(x-4)}
\end{aligned}
$$

Find $\dfrac{f(x+h)-f(x)}{h}$

$$
\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{-12h}{(x+h-4)(x-4)}\times\frac{1}{h} \\
&= \frac{-12}{(x+h-4)(x-4)}
\end{aligned}
$$

Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$
\begin{aligned}
\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{-12}{(x+h-4)(x-4)} \\
&= \frac{-12}{(x+0-4)(x-4)} \\
&= \frac{-12}{(x-4)^2}
\end{aligned}
$$

Evaluate at x = 2

$$
\frac{-12}{(x-4)^2}=\frac{-12}{(2-4)^2}=\frac{-12}{4}=-3
$$

The instantaneous rate of change when x = 2 is -1.

Exercise 3: Using the definition for the derivative, find f′(x).

$f(x) = 2x^2 + 3x$

Find f(x + h)

$f(x + h) = 2(x + h)^2 + 3(x + h)$
$f(x + h) = 2(x^2 + 2xh + h^2) + 3x + 3h$
$f(x + h) = 2x^2 + 4xh + 2h^2 + 3x + 3h$

Find f(x + h) - f(x)

$f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 3x + 3h) - (2x^2 + 3x)$
$f(x + h) - f(x) = 2x^2 + 4xh + 2h^2 + 3x + 3h - 2x^2 - 3x$
$f(x + h) - f(x) = 4xh + 2h^2 + 3h$

Find $\frac{f(x+h)-f(x)}{h}$

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{4xh+2h^2+3h}{h}\\ &= 4x+2h+3\end{aligned}$$

Find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$\begin{aligned}\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}(4x+2h+3)\\ &= 4x+2(0)+3\\ &= 4x+3\end{aligned}$$

$$\begin{aligned}f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= 4x+3\end{aligned}$$

Exercise 4: Using the definition for the derivative, find f´(x) and then evaluate it at x = -3.

$$f(x) = \frac{2}{x+4}$$

Find f(x + h)

$$f(x+h) = \frac{2}{x+h+4}$$

Find f(x + h) - f(x)

$$\begin{aligned} f(x+h) - f(x) &= \frac{2}{x+h+4} - \frac{2}{x+4} \\ &= \frac{2(x+4) - 2(x+h+4)}{(x+h+4)(x+4)} \\ &= \frac{2x+8-2x-2h-8}{(x+h+4)(x+4)} \\ &= \frac{-2h}{(x+h+4)(x+4)} \end{aligned}$$

Find $\frac{f(x+h) - f(x)}{h}$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{-2h}{(x+h+4)(x+4)} \times \frac{1}{h} \\ &= \frac{-2}{(x+h+4)(x+4)} \end{aligned}$$

Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\begin{aligned} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{-2}{(x+h+4)(x+4)} \\ &= \frac{-2}{(x+0+4)(x+4)} \\ &= \frac{-2}{(x+4)^2} \end{aligned}$$

Exercise 4 (Continued):

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \frac{-2}{(x+4)^2}$$

Evaluate the derivative f′(x) at x = 2

$$f'(x) = \frac{-2}{(x+4)^2}$$
$$f'(2) = \frac{-2}{(2+4)^2}$$
$$= \frac{-2}{6^2}$$
$$= -\frac{2}{36}$$
$$= -\frac{1}{18}$$