Review Exercise Set 24

Exercise 1: Use integration by parts to find the following integral.

$$\int x^2 \ln x \; dx$$

Exercise 2: Use integration by parts to find the following integral.

$$\int x^3 e^x \; dx$$

Exercise 3: Evaluate the following integral.

$$\int_0^1 \left(x^2 - 5x\right) e^{2x} \; dx$$

Exercise 4: Find the following integral.

$$\int (6x+3)\ln(3x)\,dx$$

Exercise 5: The reaction rate to a new drug x hours after it is administered is given by the function $r'(x) = .5xe^{-x}$. Find the total reaction over the first four hours.

Review Exercise Set 24 Answer Key

Exercise 1: Use integration by parts to find the following integral.

$$\int x^2 \ln x \; dx$$

Choose u and dv

u = ln x

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

dv = x^2 dx

$$v = \frac{1}{3}x^3$$

Rewrite the indefinite integral

$$\int x^2 \ln x \; dx = \int \ln x \; x^2 \; dx$$

Perform the substitution into the integration by parts formula and simplify

$$\begin{aligned}
\int x^2 \ln x \; dx &= uv - \int v \; dx \\
&= (\ln x)\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3\left(\frac{1}{x}\right)dx \\
&= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx \\
&= \frac{1}{3}x^3 \ln x - \frac{1}{3}\left[\frac{1}{3}x^3\right] + C \\
&= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C
\end{aligned}$$

Exercise 2: Use integration by parts to find the following integral.

$$\int x^3 e^x \; dx$$

Choose u and dv

u = x^3

$$\frac{du}{dx} = 3x^2$$

du = $3x^2$ dx

dv = e^x dx

v = e^x

Exercise 2 (Continued):

Perform the substitution into the integration by parts formula and simplify

$$\begin{aligned}\int x^3e^x\ dx &= uv - \int v\ du \\ &= \left(x^3\right)\left(e^x\right) - \int e^x\, 3x^2\ dx \\ &= x^3e^x - 3\int x^2\ e^x dx\end{aligned}$$

Repeat integration by parts on the new indefinite integral

u = x^2

$\frac{du}{dx} = 2x$

du = 2x dx

dv = e^x dx

v = e^x

$$\begin{aligned}\int x^3e^x\ dx &= x^3e^x - 3\int x^2\ e^x dx \\ &= x^3e^x - 3\left[uv - \int v\ du\right] \\ &= x^3e^x - 3\left[\left(x^2\right)\left(e^x\right) - \int e^x\ 2x\ dx\right] \\ &= x^3e^x - 3x^2e^x + 3\int e^x\ 2x\ dx \\ &= x^3e^x - 3x^2e^x + 6\int x\ e^x\ dx\end{aligned}$$

Repeat integration by parts one more time on the indefinite integral

u = x

$\frac{du}{dx} = 1$

du = dx

dv = e^x dx

v = e^x

$$\begin{aligned}\int x^3e^x\ dx &= x^3e^x - 3x^2e^x + 6\int x\ e^x\ dx \\ &= x^3e^x - 3x^2e^x + 6\left[uv - \int v\ du\right] \\ &= x^3e^x - 3x^2e^x + 6\left[\left(x\right)\left(e^x\right) - \int e^x\ dx\right] \\ &= x^3e^x - 3x^2e^x + 6xe^x - 6\int e^x\ dx \\ &= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C\end{aligned}$$

Exercise 3: Evaluate the following integral.

$$\int_0^1 \left(x^2 - 5x\right) e^{2x} dx$$

Choose u and dv

$u = x^2 - 5x$

$\frac{du}{dx} = 2x - 5$

du = (2x - 5) dx

$dv = e^{2x}\ dx$

$v = \frac{1}{2}\ e^{2x}$

Perform the substitution into the integration by parts formula and simplify

$$\begin{aligned}\int_0^1 \left(x^2 - 5x\right) e^{2x} dx &= uv - \int v\, du \\ &= \left(x^2 - 5x\right)\left(\frac{1}{2} e^{2x}\right) - \int_0^1 \frac{1}{2} e^{2x} \left(2x - 5\right) dx \\ &= \frac{1}{2}\left(x^2 - 5x\right)\left(e^{2x}\right) - \frac{1}{2}\int_0^1 \left(2x - 5\right) e^{2x} dx\end{aligned}$$

Repeat integration by parts one more time on the indefinite integral

u = 2x - 5

$\frac{du}{dx} = 2$

du = 2 dx

$dv = e^{2x}\ dx$

$v = \frac{1}{2}\ e^{2x}$

$$\begin{aligned}\int_0^1 \left(x^2 - 5x\right) e^{2x} dx &= \frac{1}{2}\left(x^2 - 5x\right)\left(e^{2x}\right) - \frac{1}{2}\left[uv - \int v\, du\right] \\ &= \frac{1}{2}\left(x^2 - 5x\right)\left(e^{2x}\right) - \frac{1}{2}\left[(2x - 5)\frac{1}{2} e^{2x} - \int_0^1 \frac{1}{2} e^{2x}\, 2dx\right] \\ &= \frac{1}{2}\left(x^2 - 5x\right)\left(e^{2x}\right) - \frac{1}{4}(2x - 5)\left(e^{2x}\right) + \frac{1}{4}\int_0^1 e^{2x}\, 2dx \\ &= \left[\frac{1}{2}\left(x^2 - 5x\right)\left(e^{2x}\right) - \frac{1}{4}(2x - 5)\left(e^{2x}\right) + \frac{1}{4} e^{2x}\right]\Bigg|_0^1\end{aligned}$$

Exercise 3 (Continued):

$$\int_0^1 \left(x^2-5x\right)e^{2x}dx = \left[\frac{1}{2}\left(1^2-5(1)\right)\left(e^{2(1)}\right)-\frac{1}{4}\left(2(1)-5\right)\left(e^{2(1)}\right)+\frac{1}{4}e^{2(1)}\right]-$$
$$\left[\frac{1}{2}\left(0^2-5(0)\right)\left(e^{2(0)}\right)-\frac{1}{4}\left(2(0)-5\right)\left(e^{2(0)}\right)+\frac{1}{4}e^{2(0)}\right]$$
$$=\left[\frac{1}{2}(-4)\left(e^2\right)-\frac{1}{4}(-3)\left(e^2\right)+\frac{1}{4}e^2\right]-$$
$$\left[\frac{1}{2}(0)\left(e^0\right)-\frac{1}{4}(-5)\left(e^0\right)+\frac{1}{4}e^0\right]$$
$$=-2e^2+\frac{3}{4}e^2+\frac{1}{4}e^2-\frac{5}{4}-\frac{1}{4}$$
$$=-e^2-\frac{3}{2}$$
$$\approx -8.89$$

Exercise 4: Find the following integral.

$$\int(6x+3)\ln(3x)\,dx$$

Choose u and dv

u = ln (3x)

$$\frac{du}{dx}=\frac{3}{3x}$$

du = $\frac{1}{x}$ dx

dv = (6x + 3) dx

v = $3x^2$ + 3x

Perform the substitution into the integration by parts formula and simplify

$$\int(6x+3)\ln(3x)\,dx = uv-\int v\,du$$
$$=\left(\ln(3x)\right)\left(3x^2+3x\right)-\int\left(3x^2+3x\right)\frac{1}{x}dx$$
$$=\left(3x^2+3x\right)\ln(3x)-\int(3x+3)\,dx$$
$$=\left(3x^2+3x\right)\ln(3x)-\left(\frac{3}{2}x^2+3x\right)+C$$
$$=\left(3x^2+3x\right)\ln(3x)-\frac{3}{2}x^2-3x+C$$

Exercise 5: The reaction rate to a new drug x hours after it is administered is given by the function $r'(x) = .5xe^{-x}$. Find the total reaction over the first four hours.

Setup the definite integral

$$\int_0^4 0.5xe^{-x}dx$$

Choose u and dv

$u = 0.5x$

$$\frac{du}{dx} = 0.5$$

$du = 0.5\ dx$

$dv = e^{-x}\ dx$

$v = -e^{-x}$

Perform the substitution into the integration by parts formula and simplify

$$\begin{aligned}
\int_0^4 0.5xe^{-x}dx &= uv - \int v\ du \\
&= (0.5x)(-e^{-x}) - \int_0^4 (-e^{-x})0.5\ dx \\
&= -0.5xe^{-x} + 0.5\int_0^4 e^{-x}\ dx \\
&= \left[-0.5xe^{-x} + 0.5(-e^{-x})\right]\Big|_0^4 \\
&= \left[-0.5xe^{-x} - 0.5e^{-x}\right]\Big|_0^4 \\
&= \left[-2e^{-4} - 0.5e^{-4}\right] - [0 - 0.5] \\
&= -2.5e^{-4} + 0.5 \\
&\approx 0.4542
\end{aligned}$$