Review Exercise Set 17

Exercise 1: Test the given polar equation for symmetry.

$r = 3 \sin 4\theta$

Exercise 2: Graph the polar equation $r = 1 - 3 \cos \theta$.

Exercise 3: Graph the polar equation $r^2 = 4 \sin 2\theta$.

Review Exercise Set 17 Answer Key

Exercise 1: Test the given polar equation for symmetry.

r = 3 sin 4θ

Identify the type of polar equation

The polar equation is in the form of a rose curve, r = a sin nθ. Since n is an even integer, the rose will have 2n petals. So the graph for this equation should have 8 petals (2n = 2(4) = 8).

Test for symmetry

Polar axis	$\theta = \frac{\pi}{2}$	Pole
r = 3 sin 4 θ	r = 3 sin 4 θ	r = 3 sin 4 θ
r = 3 sin 4 (-θ)	(-r) = 3 sin 4 (-θ)	(-r) = 3 sin 4 θ
r = 3 sin (-4 θ)	-r = -3 sin 4 θ	r = -3 sin 4 θ
r = -3 sin 4 θ	r = 3 sin 4 θ	
Fails test	Passes test	Fails test

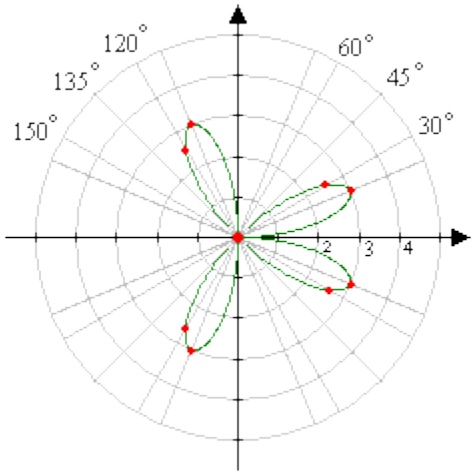
Evaluate r at different values of θ

θ	r = 3 sin 4 θ	(r, θ)
0°	r = 3 sin 4(0°) = 0	(0, 0°)
22.5°	r = 3 sin 4(22.5°) = 3	(3, 22.5°)
30°	r = 3 sin 4(30°) = $\frac{3\sqrt{3}}{2}$	($\frac{3\sqrt{3}}{2}$, 30°)
45°	r = 3 sin 4(45°) = 0	(0, 45°)
60°	r = 3 sin 4(60°) = -$\frac{3\sqrt{3}}{2}$	(-$\frac{3\sqrt{3}}{2}$, 60°)
67.5°	r = 3 sin 4(67.5°) = -3	(-3, 67.5°)
90°	r = 3 sin 4(90°) = 0	(0, 90°)
112.5°	r = 3 sin 4(112.5°) = 3	(3, 112.5°)
120°	r = 3 sin 4(120°) = $\frac{3\sqrt{3}}{2}$	($\frac{3\sqrt{3}}{2}$, 120°)
135°	r = 3 sin 4(135°) = 0	(0, 135°)

Exercise 1 (Continued):

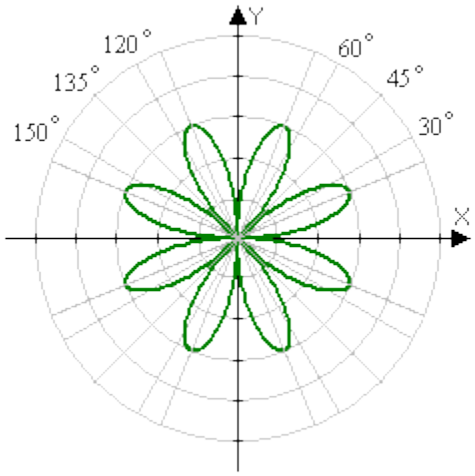
150°	r = 3 sin 4(150) = $-\frac{3\sqrt{3}}{2}$	$(-\frac{3\sqrt{3}}{2}, 150°)$
157.5°	r = 3 sin 4(157.5°) = -3	(-3, 157.5°)
180°	r = 3 sin 4(180°) = 0	(0, 180°)

Plot the points



Use the symmetry to complete the graph

Reflect the points across the line $\theta = \frac{\pi}{2}$



Exercise 2: Graph the polar equation r = 1 - 3 cos θ.

Identify the type of polar equation

The polar equation is in the form of a limacon, r = a - b cos θ. Since the ratio of a over b is less than one, it will have both an inner and outer loop. The loops will be along the polar axis since the function is cosine and will loop to the left since the sign between a and b is minus.

Test for symmetry

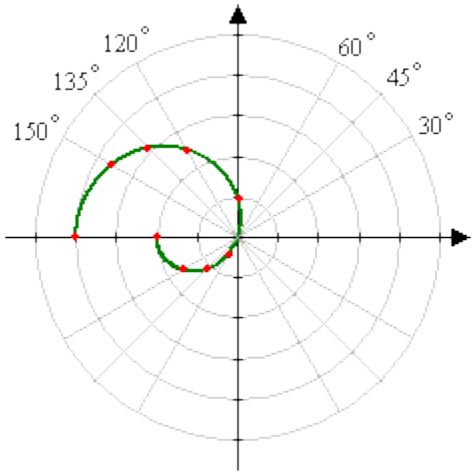
Polar axis	$\theta = \frac{\pi}{2}$	Pole
r = 1 - 3 cos θ	r = 1 - 3 cos θ	r = 1 - 3 cos θ
r = 1 - 3 cos (-θ)	(-r) = 1 - 3 cos (-θ)	(-r) = 1 - 3 cos θ
r = 1 - 3 cos θ	-r = 1 - 3 cos θ	r = -1 + 3 cos θ
	r = -1 + 3 cos θ	
Passes test	Fails test	Fails test

Evaluate r at different values of θ

θ	r = 1 - 3 cos θ	(r, θ)
0°	r = 1 - 3 cos (0°) = -2	(-2, 0°)
30°	r = 1 - 3 cos (30°) = $\frac{2-3\sqrt{3}}{2}$	($\frac{2-3\sqrt{3}}{2}$, 30°)
45°	r = 1 - 3 cos (45°) = $\frac{2-3\sqrt{2}}{2}$	($\frac{2-3\sqrt{2}}{2}$, 45°)
60°	r = 1 - 3 cos (60°) = $-\frac{1}{2}$	($-\frac{1}{2}$, 60°)
90°	r = 1 - 3 cos (90°) = 1	(1, 90°)
120°	r = 1 - 3 cos (120°) = $\frac{5}{2}$	($\frac{5}{2}$, 120°)
135°	r = 1 - 3 cos (135°) = $\frac{2+3\sqrt{2}}{2}$	($\frac{2+3\sqrt{2}}{2}$, 135°)
150°	r = 1 - 3 cos (150°) = $\frac{2+3\sqrt{3}}{2}$	($\frac{2+3\sqrt{3}}{2}$, 150°)
180°	r = 1 - 3 cos (180°) = 4	(4, 180°)

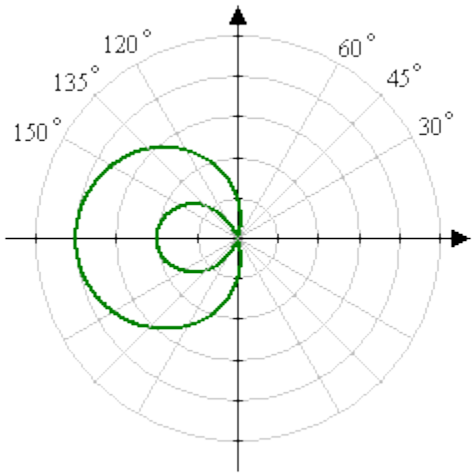
Exercise 2 (Continued):

Plot the points



Use the symmetry to complete the graph

Reflect the points across the polar axis



Exercise 3: Graph the polar equation $r^2 = 4 \sin 2\theta$.

Identify the type of polar equation

The polar equation is in the form of a lemniscate, $r^2 = a^2 \sin 2\theta$.

Test for symmetry

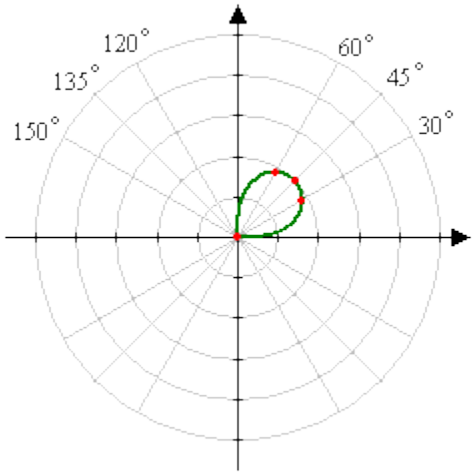
Polar axis	$\theta = \frac{\pi}{2}$	Pole
$r^2 = 4 \sin 2\theta$	$r^2 = 4 \sin 2\theta$	$r^2 = 4 \sin 2\theta$
$r^2 = 4 \sin 2(-\theta)$	$(-r)^2 = 4 \sin 2(-\theta)$	$(-r)^2 = 4 \sin 2\theta$
$r^2 = 4 \sin (-2\theta)$	$r^2 = 4 \sin (-2\theta)$	$r^2 = 4 \sin 2\theta$
$r^2 = -4 \sin 2\theta$	$r^2 = -4 \sin 2\theta$	
Fails test	Fails test	Passes test

Evaluate r at different values of θ

θ	$r^2 = 4 \sin 2\theta$	(r, θ)
0°	$r^2 = 4 \sin 2(0°) = 0$ $r = 0$	(0, 0°)
30°	$r^2 = 4 \sin 2(30°) = 2\sqrt{3}$ $r = \pm\sqrt[4]{12}$	$(\sqrt[4]{12}, 30°)$ $(-\sqrt[4]{12}, 30°)$
45°	$r^2 = 4 \sin 2(45°) = 4$ $r = \pm 2$	(2, 45°) (-2, 45°)
60°	$r^2 = 4 \sin 2(60°) = 2\sqrt{3}$ $r = \pm\sqrt[4]{12}$	$(\sqrt[4]{12}, 60°)$ $(-\sqrt[4]{12}, 60°)$
90°	$r^2 = 4 \sin 2(90°) = 0$ $r = 0$	(0, 90°)

Exercise 3 (Continued):

Plot the points



Use the symmetry to complete the graph

Reflect the points across the pole