Review Exercise Set 19

Exercise 1: Write the vector v, with its initial point at (-2, 3) and its terminal point at (6, 0), in terms of i and j. Also, find the magnitude of the vector.

Exercise 2: Find $v + w$, if $v = 2i + 10j$ and $w = -4i + 6j$.

Exercise 3: Find $2v - 3w$, if $v = 7i - 2j$ and $w = 3i + 7j$.

Exercise 4: Find the unit vector that has the same direction as $v = 6i - 11j$.

Exercise 5: A ship is traveling at 18 knots on a bearing of N15E. There is a strong water current flowing at 7 knots from the northwest on a bearing of S80E. What is the actual course and speed of the ship?

Review Exercise Set 19 Answer Key

Exercise 1: Write the vector ***v***, with its initial point at (-2, 3) and its terminal point at (6, 0), in terms of ***i*** and ***j***. Also, find the magnitude of the vector.

Write the vector

$(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (6, 0)$

$\boldsymbol{v} = (x_2 - x_1)\boldsymbol{i} + (y_2 - y_1)\boldsymbol{j}$

$\boldsymbol{v} = (6 - (-2))\boldsymbol{i} + (0 - 3)\boldsymbol{j}$
$\boldsymbol{v} = (6 + 2)\boldsymbol{i} + (0 - 3)\boldsymbol{j}$
$\boldsymbol{v} = 8\boldsymbol{i} - 3\boldsymbol{j}$

Find the magnitude

$$\begin{aligned}\|v\| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6-(-2))^2 + (0-3)^2} \\ &= \sqrt{(8)^2 + (-3)^2} \\ &= \sqrt{64+9} \\ &= \sqrt{73}\end{aligned}$$

Exercise 2: Find $\boldsymbol{v} + \boldsymbol{w}$, if $\boldsymbol{v} = 2\boldsymbol{i} + 10\boldsymbol{j}$ and $\boldsymbol{w} = -4\boldsymbol{i} + 6\boldsymbol{j}$.

$\boldsymbol{v} + \boldsymbol{w} = (2\boldsymbol{i} + 10\boldsymbol{j}) + (-4\boldsymbol{i} + 6\boldsymbol{j})$
$\boldsymbol{v} + \boldsymbol{w} = (2 + (-4))\boldsymbol{i} + (10 + 6)\boldsymbol{j}$
$\boldsymbol{v} + \boldsymbol{w} = -2\boldsymbol{i} + 16\boldsymbol{j}$

Exercise 3: Find $2\boldsymbol{v} - 3\boldsymbol{w}$, if $\boldsymbol{v} = 7\boldsymbol{i} - 2\boldsymbol{j}$ and $\boldsymbol{w} = 3\boldsymbol{i} + 7\boldsymbol{j}$.

Find $2\boldsymbol{v}$

$2\boldsymbol{v} = 2(7\boldsymbol{i} - 2\boldsymbol{j})$
$2\boldsymbol{v} = 14\boldsymbol{i} - 4\boldsymbol{j}$

Find $3\boldsymbol{w}$

$3\boldsymbol{w} = 3(3\boldsymbol{i} + 7\boldsymbol{j})$
$3\boldsymbol{w} = 9\boldsymbol{i} + 21\boldsymbol{j}$

Exercise 3 (Continued):

Find $2\boldsymbol{v} - 3\boldsymbol{w}$

$2\boldsymbol{v} - 3\boldsymbol{w} = (14\boldsymbol{i} - 4\boldsymbol{j}) - (9\boldsymbol{i} + 21\boldsymbol{j})$
$2\boldsymbol{v} - 3\boldsymbol{w} = (14\boldsymbol{i} - 4\boldsymbol{j}) + (-1)(9\boldsymbol{i} + 21\boldsymbol{j})$
$2\boldsymbol{v} - 3\boldsymbol{w} = (14\boldsymbol{i} - 4\boldsymbol{j}) + (-9\boldsymbol{i} - 21\boldsymbol{j})$
$2\boldsymbol{v} - 3\boldsymbol{w} = (14 + (-9))\boldsymbol{i} + (-4 + (-21))\boldsymbol{j}$
$2\boldsymbol{v} - 3\boldsymbol{w} = (14 - 9)\boldsymbol{i} + (-4 - 21)\boldsymbol{j}$
$2\boldsymbol{v} - 3\boldsymbol{w} = 5\boldsymbol{i} - 25\boldsymbol{j}$

Exercise 4: Find the unit vector that has the same direction as $\boldsymbol{v} = 6\boldsymbol{i} - 11\boldsymbol{j}$.

Find the magnitude of $\boldsymbol{v}$

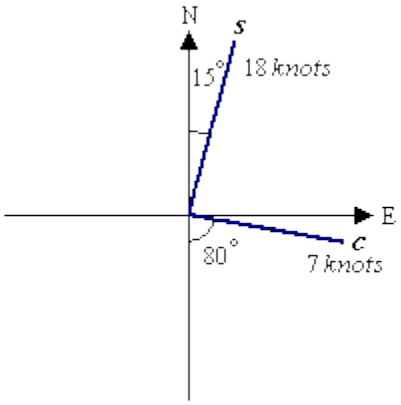
$$\begin{aligned}\|v\| &= \sqrt{a^2 + b^2} \\ &= \sqrt{(6)^2 + (-11)^2} \\ &= \sqrt{36 + 121} \\ &= \sqrt{157}\end{aligned}$$

Divide the vector by its magnitude

$$\begin{aligned}\frac{v}{\|v\|} &= \frac{a}{\|v\|}i + \frac{b}{\|v\|}j \\ &= \frac{6}{\sqrt{157}}i + \frac{-11}{\sqrt{157}}j \\ &= \frac{6}{\sqrt{157}}i - \frac{11}{\sqrt{157}}j\end{aligned}$$

Exercise 5: A ship is traveling at 18 knots on a bearing of N15°E. There is a strong water current flowing at 7 knots from the northwest on a bearing of S80°E. What is the actual course and speed of the ship? Round values to the nearest tenth.

Draw diagram of the problem using vectors



vector ***s*** will represent the speed and direction of the ship
vector ***c*** will represent the speed and direction of the current

Find the angle between the vectors and the x-axis (due East)

$$\theta_s = 90° - 15° = 75°$$
$$\theta_c = 90° - 80° = 10°$$

Find the vector components of vector ***s***

$$a = \|s\|\cos\theta_s \qquad b = \|s\|\sin\theta_s$$
$$= \|18\|\cos 75° \qquad = \|18\|\sin 75°$$
$$a \approx 4.7 \qquad b \approx 17.4$$

s ≈ 4.7***i*** + 17.4***j***

Find the vector components of vector ***c***

(since ***c*** is in quadrant IV b must be negative)

$$a = \|c\|\cos\theta_c \qquad b = -\|c\|\sin\theta_c$$
$$= \|7\|\cos 10° \qquad = -\|7\|\sin 10°$$
$$a \approx 6.9 \qquad b \approx -1.2$$

c ≈ 6.9***i*** - 1.2***j***

Exercise 5 (Continued):

Find the resultant vector of $\boldsymbol{s} + \boldsymbol{c}$

$\boldsymbol{s} + \boldsymbol{c} \approx (4.7\boldsymbol{i} + 17.4\boldsymbol{j}) + (6.9\boldsymbol{i} - 1.2\boldsymbol{j})$
$\boldsymbol{s} + \boldsymbol{c} \approx (4.7 + 6.9)\boldsymbol{i} + (17.4 - 1.2)\boldsymbol{j}$
$\boldsymbol{s} + \boldsymbol{c} \approx 11.6\boldsymbol{i} + 16.2\boldsymbol{j}$

Find the magnitude of the resultant vector

$$\begin{aligned} \|s+c\| &= \sqrt{a^2+b^2} \\ &\approx \sqrt{(11.6)^2+(16.2)^2} \\ &\approx \sqrt{135.46+262.44} \\ &\approx \sqrt{397} \\ &\approx 19.9 \end{aligned}$$

Find the angle between the resultant vector and the x-axis

$$\begin{aligned} \theta_r &= \tan^{-1}\frac{16.2}{11.6} \\ &\approx 54.4^\circ \end{aligned}$$

bearing ≈ 90° - 54.4° ≈ 35.6°

The ship is actually traveling 19.9 knots on a bearing of N35.6°E.