Review Exercise Set 11

Exercise 1: Find the exact value of cos 2x under the given conditions.

$$\sin x = \frac{12}{13} \textit{ and } x \textit{ lies in quadrant } II$$

Exercise 2: Find the exact value of the given expression by first rewriting it as the sine, cosine, or tangent of a double angle.

$$\cos^2 135° - \sin^2 135°$$

Exercise 3: Verify the given identity.

$$\sin(3x) = -4\sin^3 x + 3\sin x$$

Exercise 4: Use the power-reducing formulas to rewrite the given expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1.

$$8\sin^4 x$$

Exercise 5: Verify the given identity.

$$\left(\tan\frac{x}{2}\right)(1+\cos x)=\sin x$$

Review Exercise Set 11 Answer Key

Exercise 1: Find the exact value of cos 2x under the given conditions.

$$\sin x = \frac{12}{13} \; and \; x \; lies \; in \; quadrant \; II$$

Use one of the double angle identities for cos 2x

$$\begin{aligned}\cos 2x &= 1 - 2\sin^2 x \\ &= 1 - 2\left(\frac{12}{13}\right)^2 \\ &= 1 - 2\left(\frac{144}{169}\right) \\ &= 1 - \frac{288}{169} \\ &= -\frac{119}{169}\end{aligned}$$

Exercise 2: Find the exact value of the given expression by first rewriting it as the sine, cosine, or tangent of a double angle.

$$\cos^2 135° - \sin^2 135°$$

Use the cos 2x identity

$$\begin{aligned}\cos^2 x - \sin^2 x &= \cos 2x \\ \cos^2 135° - \sin^2 135° &= \cos 2(135°) \\ &= \cos 270° \\ &= 0\end{aligned}$$

Exercise 3: Verify the given identity.

$$\sin(3x) = -4\sin^3 x + 3\sin x$$

Rewrite 3x as a sum of two terms

$$\sin(2x + x) = -4\sin^3 x + 3\sin x$$

Use the sum identity for sin

$$\sin 2x \cos x + \cos 2x \sin x = -4\sin^3 x + 3\sin x$$

Exercise 3 (Continued):

Use the double angle identities for sin and cos

$$(2\sin x\cos x)\cos x+(1-2\sin^2 x)\sin x=-4\sin^3 x+3\sin x$$
$$2\sin x\cos^2 x+\sin x-2\sin^3 x=-4\sin^3 x+3\sin x$$

Use the Pythagorean identity

$$2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x=-4\sin^3 x+3\sin x$$
$$2\sin x-2\sin^3 x+\sin x-2\sin^3 x=-4\sin^3 x+3\sin x$$

Combine like terms

$$-4\sin^3 x+3\sin x=-4\sin^3 x+3\sin x$$

Exercise 4: Use the power-reducing formulas to rewrite the given expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1..

$$8\sin^4 x$$

Rewrite as $\sin^2$ x to another power

$$8\sin^4 x=8(\sin^2 x)^2$$

Use the power reducing identity for $\sin^2$ x

$$8\sin^4 x=8\left(\frac{1-\cos 2x}{2}\right)^2$$

Simplify

$$8\sin^4 x=8\left(\frac{1-2\cos 2x+\cos^2 2x}{4}\right)$$
$$=2(1-2\cos 2x+\cos^2 2x)$$
$$=2-4\cos 2x+2\cos^2 2x$$

Exercise 4 (Continued):

Use the power reducing identity for $\cos^2 x$

$$8\sin^4 x = 2 - 4\cos 2x + 2\left(\frac{1+\cos 2(2x)}{2}\right)$$

Simplify

$$\begin{aligned} 8\sin^4 x &= 2 - 4\cos 2x + 1 + \cos(4x) \\ &= 3 - 4\cos 2x + \cos(4x) \end{aligned}$$

Exercise 5: Verify the given identity.

$$\left(\tan\frac{x}{2}\right)(1+\cos x) = \sin x$$

Use the half angle identity for tangent

$$\left(\frac{\sin x}{1+\cos x}\right)(1+\cos x) = \sin x$$

$$\sin x = \sin x$$