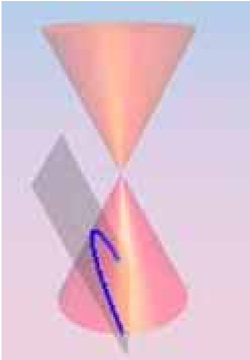
PARABOLA

This section will discuss the concepts of conic sections and focus specifically on parabolas. A conic section may be defined as the intersection of a plane and a cone. The curves that are defined by these intersections are known as conic sections. Should a plane not cut completely through a cone, at an angle not perpendicular to its axis, the curve is known as a parabola (see figure below).



The standard equation of the parabola is based on the axis of the parabola. If the parabola opens to the left or right (along the x-axis) with its vertex at the origin, its equation is $y^2 = 4ax$. If the parabola opens up or down (along the y-axis) with its vertex at the origin, its equation will be $x^2 = 4ay$.

Two further elements are needed to complete the analysis of parabolas, the directrix, and the focus. With a vertical axis the directrix is $y = -a$, while a horizontal axis has a directrix of $x = -a$. The focus of the equation is found by manipulating the equation into the form $x^2 = 4ay$ or $y^2 = 4ax$, where $a \neq 0$ for either, to solve for a.

The examples that follow will show how to determine the focus and directrix of a parabola and then how to determine the equation of a parabola from either a focus or directrix.

Example 1: Locate the focus and directrix then graph the equation $y^2 = 4x$.

Solution:

Step 1: Analysis.

Since the equation has its vertex at the origin and has a horizontal axis, the equation of its directrix is $x = -a$ and a focus found by using $y^2 = 4ax$.

Step 2: Solve for the focus.

The elements of the problem are substituted into the formula to solve for the focus.

$$y^2 = 4ax$$
$$y^2 = 4(1)x$$
$$\therefore \text{ Focus} = (1, 0)$$

Step 3: Solve for the directrix.

The elements of the problem are substituted into the formula to solve for the directrix, using the solution for *a* found in step 2.

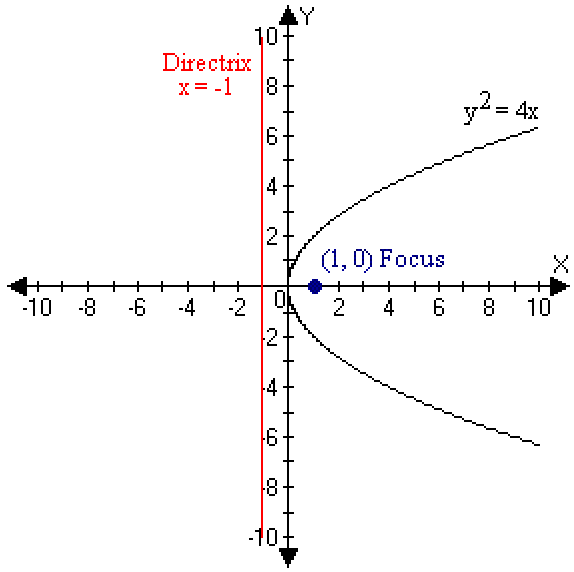
$$x = -a$$
$$x = -(1) = (-1)$$
$$\therefore \text{ Directrix} = x = (-1)$$

Step 4: Plot points and graph.

Several points are created to plot the graph of the equation.

x	0	1	4
y	0	2,-2	4,-4

Example 1 (Continued):



Example 2: Locate the focus and directrix then graph the equation $x^2 = 8y$.

Solution:

Step 1: Analysis.

Since the equation has its vertex at the origin and has a vertical axis, the equation of its directrix is $y = -a$ and a focus found by using $x^2 = 4ay$.

Step 2: Solve for the focus.

The elements of the problem are substituted into the formula to solve for the focus.

$x^2 = 4ay$
$x^2 = 8y$
$x^2 = 4(2)y$
$\therefore$ Focus = (0, 2)

Example 2 (Continued):

Step 3: Solve for the directrix.

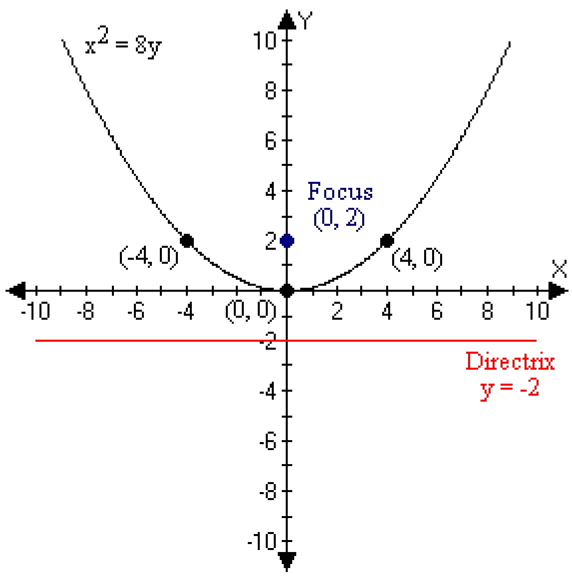
The elements of the problem are substituted into the formula to solve for the directrix, using the solution for *a* found in step 2.

$y = -a$
$y = -(2) = (-2)$
$\therefore$ Directrix = $y = (-2)$

Step 4: Plot points and graph.

Several points are created to plot the graph of the equation.

x	0	4,-4
y	0	2



Example 3: Find the directrix and the equation of the parabola whose origin is the vertex and whose focus is (– 6, 0). Graph and label.

Solution:

Step 1: Analysis.

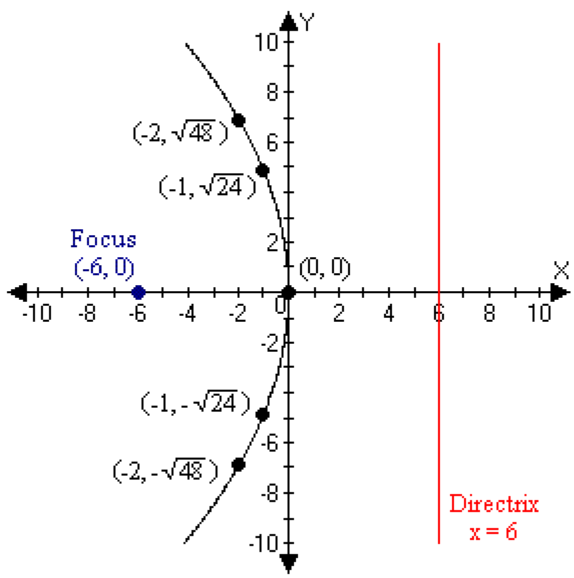
Since the focus is given to be (– 6, 0) and the vertex is at the origin, it may be determined that the axis of the equation is horizontal since it must pass through both the vertex and the focus. This means that the formula of the directrix and equation will be $y^2 = 4ax$.

Step 2: Solve.

Since the focus is given to be (– 6, 0), $a = -6$. The equation of the parabola will be $y^2 = 4(-6)x = -24x$ and the directrix will be x = – (– 6) = 6.

Step 3: Plot and graph.

x	0	-1	-2
y	0	$\pm\sqrt{24} = \pm 2\sqrt{6}$	$\pm\sqrt{48} = \pm 4\sqrt{3}$



Example 4: Find the focus and equation of the parabola whose directrix is $y = -5$ and vertex is the origin.

Solution:

Step 1: Analysis.

Since the directrix is given to be $y = -5$ (a horizontal line) and the directrix is perpendicular to the axis of the parabola, the formula for the equation will be $x^2 = 4ay$ and the focus will be $(0, -a)$.

Step 2: Solve.

The focus of the parabola is solved for by letting $-a = y$. This will yield $-(-5) = 5 = y$. The focus is therefore (0, 5). These values are substituted into the formula for a parabola whose axis is vertical to give $x^2 = 4ay = 4(5)y = 20y$.

Step 3: Plot and graph.

x	0	$\pm\sqrt{20} = 2\sqrt{5}$	$\pm\sqrt{40} = 2\sqrt{10}$
y	0	1	2