Review Exercise Set 24

Exercise 1: For the given statement S_n write the statement for S_1 to show that it is true.

$$S_n: 1 + 4 + 7 + 10 + 13 + \ldots + (3n - 2) = \frac{n(3n-1)}{2}$$

Exercise 2: For the given statement S_n write the statement S_{k+1} and simplify completely.

$$S_n: 2 + 4 + 6 + 8 + 10 + \ldots + 2n = n(n + 1)$$

Exercise 3: Use mathematical induction to prove that the given statement is true for all positive integers n.

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2\left(n+1\right)^2}{4}$$

Exercise 4: Use the principle of mathematical induction to prove that the given statement is true for every positive integer n.

$n^3 - n + 3$ is divisible by 3

Review Exercise Set 24 Answer Key

Exercise 1: For the given statement S_n write the statement for S_1 to show that it is true.

S_n: 1 + 4 + 7 + 10 + 13 + ... + (3n - 2) = $\frac{n(3n-1)}{2}$

Substitute 1 for n on the right side of the equation and show it equals the first term of 1

S_1:

1 = $\frac{1(3(1)-1)}{2}$

1 = $\frac{1(2)}{2}$

1 = 1

Exercise 2: For the given statement S_n write the statement S_{k+1} and simplify completely.

S_n: 2 + 4 + 6 + 8 + 10 + ... + 2n = n(n + 1)

Substitute k + 1 in for n and simplify

S_{k+1}:

2 + 4 + 6 + 8 + 10 + ... + 2(k+1) = (k + 1)[(k + 1) + 1]
2 + 4 + 6 + 8 + 10 + ... + 2(k+1) = (k + 1)(k + 2)

Exercise 3: Use mathematical induction to prove that the given statement is true for all positive integers n.

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

Show S_1 is true

1 = $\frac{(1)^2((1)+1)^2}{4}$

1 = $\frac{(1)^2(2)^2}{4}$

1 = $\frac{(1)(4)}{4}$

1 = 1

Exercise 3 (Continued):

Show that if S_k is true then S_{k+1} is true

S_k:

$$1^3 + 2^3 + 3^3 + ... + k^3 = \frac{k^2(k+1)^2}{4}$$

S_{k+1}:

$$\begin{aligned} 1^3 + 2^3 + 3^3 + ... + (k+1)^3 &= \frac{(k+1)^2((k+1)+1)^2}{4} \\ &= \frac{(k+1)^2(k+2)^2}{4} \end{aligned}$$

Start with S_k and add $(k + 1)^3$ to both sides

$$1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$$

Simplify the left side to show that it is equal to $\frac{(k+1)^2(k+2)^2}{4}$

$$\begin{aligned} 1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3 &= \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4} \\ &= \frac{k^2(k+1)^2 + 4(k+1)^3}{4} \\ &= \frac{(k+1)^2\left[k^2 + 4(k+1)\right]}{4} \\ &= \frac{(k+1)^2(k^2 + 4k + 4)}{4} \\ &= \frac{(k+1)^2(k+2)^2}{4} \end{aligned}$$

Exercise 4: Use the principle of mathematical induction to prove that the given statement is true for every positive integer n.

$$n^3 - n + 3 \text{ is divisible by } 3$$

Show S_1 is divisible by 3

If a number is divisible by 3 then it can be written as the product of 3 and some positive integer (p).

$n^3 - n + 3 = 3p$
$(1)^3 - (1) + 3 = 3p$
$1 - 1 + 3 = 3p$
$3 = 3p$

3 is divisible by 3 so the statement is true for n = 1

Show that if S_k is divisible by 3 then S_{k+1} is also divisible by 3

S_k:

$$k^3 - k + 3 = 3p$$

S_{k+1}:

$(k + 1)^3 - (k + 1) + 3 = 3q$; where q is a positive integer

Start with the left hand side and simplify

$$(k + 1)^3 - (k + 1) + 3 = k^3 + 3k^2 + 3k + 1 - k - 1 + 3$$

group the terms $k^3 - k + 3$ together since we know it equals 3p

$$(k + 1)^3 - (k + 1) + 3 = (k^3 - k + 3) + 3k^2 + 3k + 1 - 1$$

Substitute in 3p and combine like terms

$$(k + 1)^3 - (k + 1) + 3 = 3p + 3k^2 + 3k$$

Factor out the common terms of 3

$$(k + 1)^3 - (k + 1) + 3 = 3(p + k^2 + k)$$

Let $q = p + k^2 + k$

$$(k + 1)^3 - (k + 1) + 3 = 3q$$