

Review Exercise Set 5

Exercise 1: Determine the amplitude and period of the given sine function and then graph it over one complete period.

$$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}x\right)$$

Exercise 2: Determine the amplitude and period of the given cosine function and then graph it over one complete period.

$$y = 3 \cos(4x)$$

Exercise 3: Determine the amplitude, period, and phase shift of the given trigonometric function and then graph it over one complete period.

$$4y = 3 \sin\left(2x - \frac{\pi}{2}\right)$$

Exercise 4: Graph the given trigonometric function over one complete period.

$$y = -\sin\left(x + \frac{2\pi}{3}\right)$$

Exercise 5: Graph the given trigonometric function over one complete period.

$$y = 2\cos(2\pi x)$$

Review Exercise Set 5 Answer Key

Exercise 1: Determine the amplitude and period of the given sine function and then graph it over one complete period.

$$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}x\right)$$

Find the amplitude

$$\begin{aligned} \text{amplitude} &= |A| \\ &= \left| -\frac{5}{3} \right| \\ &= \frac{5}{3} \end{aligned}$$

Find the period

$$\begin{aligned} \text{period} &= \frac{2\pi}{B} \\ &= \frac{2\pi}{\frac{2\pi}{3}} \\ &= \frac{2\pi}{1} \times \frac{3}{2\pi} \\ &= 3 \end{aligned}$$

Find the x-values of the key points of the period

$$\begin{aligned} \text{interval width} &= \frac{\text{period}}{4} \\ &= \frac{3}{4} \end{aligned}$$

There is no phase shift so our first key point will start at $x = 0$ and increase by $\frac{3}{4}$

$$x_1 = 0 \qquad x_2 = 0 + \frac{3}{4} \qquad x_3 = \frac{3}{4} + \frac{3}{4} \qquad x_4 = \frac{3}{2} + \frac{3}{4} \qquad x_5 = \frac{9}{4} + \frac{3}{4}$$

$$x_1 = 0 \qquad x_2 = \frac{3}{4} \qquad x_3 = \frac{3}{2} \qquad x_4 = \frac{9}{4} \qquad x_5 = 3$$

Exercise 1 (Continued):

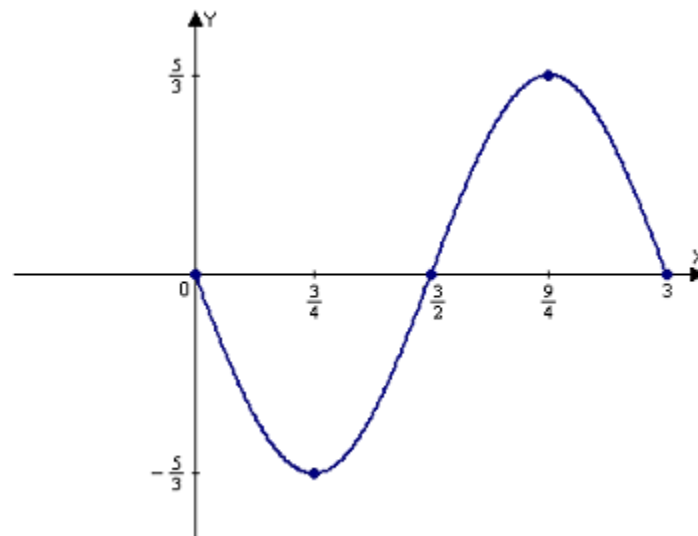
Find the y-values of the key points of the period

x-value	y-value	key point
0	$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}(0)\right)$ $= -\frac{5}{3} \sin(0)$ $= -\frac{5}{3}(0)$ $= 0$	(0, 0)
$\frac{3}{4}$	$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}\left(\frac{3}{4}\right)\right)$ $= -\frac{5}{3} \sin\left(\frac{\pi}{2}\right)$ $= -\frac{5}{3}(1)$ $= -\frac{5}{3}$	$\left(\frac{3}{4}, -\frac{5}{3}\right)$
$\frac{3}{2}$	$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}\left(\frac{3}{2}\right)\right)$ $= -\frac{5}{3} \sin(\pi)$ $= -\frac{5}{3}(0)$ $= 0$	$\left(\frac{3}{2}, 0\right)$
$\frac{9}{4}$	$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}\left(\frac{9}{4}\right)\right)$ $= -\frac{5}{3} \sin\left(\frac{3\pi}{2}\right)$ $= -\frac{5}{3}(-1)$ $= \frac{5}{3}$	$\left(\frac{9}{4}, \frac{5}{3}\right)$

Exercise 1 (Continued):

x-value	y-value	key point
3	$y = -\frac{5}{3} \sin\left(\frac{2\pi}{3}(3)\right)$ $= -\frac{5}{3} \sin(2\pi)$ $= -\frac{5}{3}(0)$ $= 0$	(3, 0)

Graph the function



Exercise 2: Determine the amplitude and period of the given cosine function and then graph it over one complete period.

$$y = 3 \cos(4x)$$

Find the amplitude

$$\begin{aligned} \text{amplitude} &= |A| \\ &= |3| \\ &= 3 \end{aligned}$$

Exercise 2 (Continued):

Find the period

$$\begin{aligned} \text{period} &= \frac{2\pi}{B} \\ &= \frac{2\pi}{4} \\ &= \frac{\pi}{2} \end{aligned}$$

Find the x-values of the key points of the period

$$\begin{aligned} \text{interval width} &= \frac{\text{period}}{4} \\ &= \frac{\frac{\pi}{2}}{4} \\ &= \frac{\pi}{2} \times \frac{1}{4} \\ &= \frac{\pi}{8} \end{aligned}$$

There is no phase shift so our first key point will start at $x = 0$ and increase by $\frac{\pi}{8}$

$$x_1 = 0 \quad x_2 = 0 + \frac{\pi}{8} \quad x_3 = \frac{\pi}{8} + \frac{\pi}{8} \quad x_4 = \frac{\pi}{4} + \frac{\pi}{8} \quad x_5 = \frac{3\pi}{8} + \frac{\pi}{8}$$

$$x_1 = 0 \quad x_2 = \frac{\pi}{8} \quad x_3 = \frac{\pi}{4} \quad x_4 = \frac{3\pi}{8} \quad x_5 = \frac{\pi}{2}$$

Find the y-values of the key points of the period

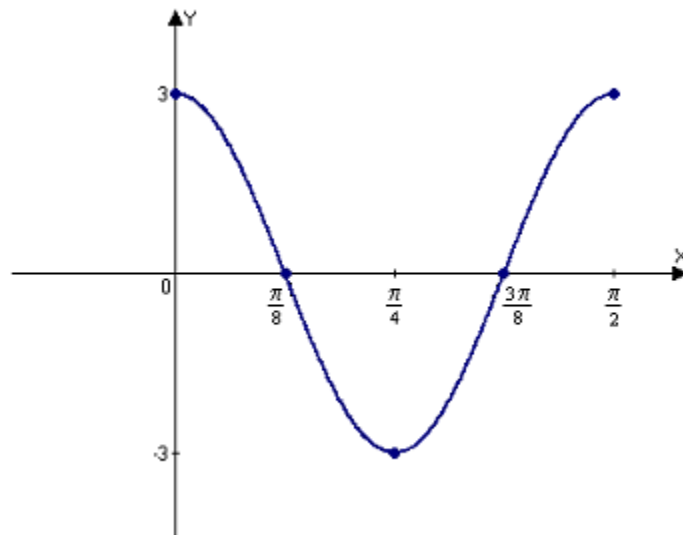
x-value	y-value	key point
0	$\begin{aligned} y &= 3\cos(4(0)) \\ &= 3\cos(0) \\ &= 3(1) \\ &= 3 \end{aligned}$	(0, 3)

Exercise 2 (Continued):

x-value	y-value	key point
$\frac{\pi}{8}$	$y = 3 \cos \left(4 \left(\frac{\pi}{8} \right) \right)$ $= 3 \cos \left(\frac{\pi}{2} \right)$ $= 3(0)$ $= 0$	$\left(\frac{\pi}{8}, 0 \right)$
$\frac{\pi}{4}$	$y = 3 \cos \left(4 \left(\frac{\pi}{4} \right) \right)$ $= 3 \cos (\pi)$ $= 3(-1)$ $= -3$	$\left(\frac{\pi}{4}, -3 \right)$
$\frac{3\pi}{8}$	$y = 3 \cos \left(4 \left(\frac{3\pi}{8} \right) \right)$ $= 3 \cos \left(\frac{3\pi}{2} \right)$ $= 3(0)$ $= 0$	$\left(\frac{3\pi}{8}, 0 \right)$
$\frac{\pi}{2}$	$y = 3 \cos \left(4 \left(\frac{\pi}{2} \right) \right)$ $= 3 \cos (2\pi)$ $= 3(1)$ $= 3$	$\left(\frac{\pi}{2}, 3 \right)$

Exercise 2 (Continued):

Graph the function



Exercise 3: Determine the amplitude, period, and phase shift of the given trigonometric function and then graph it over one complete period.

$$4y = 3 \sin\left(2x - \frac{\pi}{2}\right)$$

Find the amplitude

$$4y = 3 \sin\left(2x - \frac{\pi}{2}\right)$$

$$y = \frac{3}{4} \sin\left(2x - \frac{\pi}{2}\right)$$

$$\begin{aligned} \text{amplitude} &= |A| \\ &= \left|\frac{3}{4}\right| \\ &= \frac{3}{4} \end{aligned}$$

Exercise 3 (Continued):

Find the period

$$\begin{aligned} \text{period} &= \frac{2\pi}{B} \\ &= \frac{2\pi}{2} \\ &= \pi \end{aligned}$$

Find the phase shift

$$\begin{aligned} \text{phase shift} &= \frac{C}{B} \\ &= \frac{\frac{\pi}{2}}{2} \\ &= \frac{\pi}{2} \times \frac{1}{2} \\ &= \frac{\pi}{4} \end{aligned}$$

Find the x-values of the key points of the period

$$\begin{aligned} \text{interval width} &= \frac{\text{period}}{4} \\ &= \frac{\pi}{4} \end{aligned}$$

Since this function has a phase shift, the first key point will start at $x = \frac{\pi}{4}$ and increase

by $\frac{\pi}{4}$

$$x_1 = \frac{\pi}{4} \quad x_2 = \frac{\pi}{4} + \frac{\pi}{4} \quad x_3 = \frac{\pi}{2} + \frac{\pi}{4} \quad x_4 = \frac{3\pi}{4} + \frac{\pi}{4} \quad x_5 = \pi + \frac{\pi}{4}$$

$$x_1 = \frac{\pi}{4} \quad x_2 = \frac{\pi}{2} \quad x_3 = \frac{3\pi}{4} \quad x_4 = \pi \quad x_5 = \frac{5\pi}{4}$$

Exercise 3 (Continued):

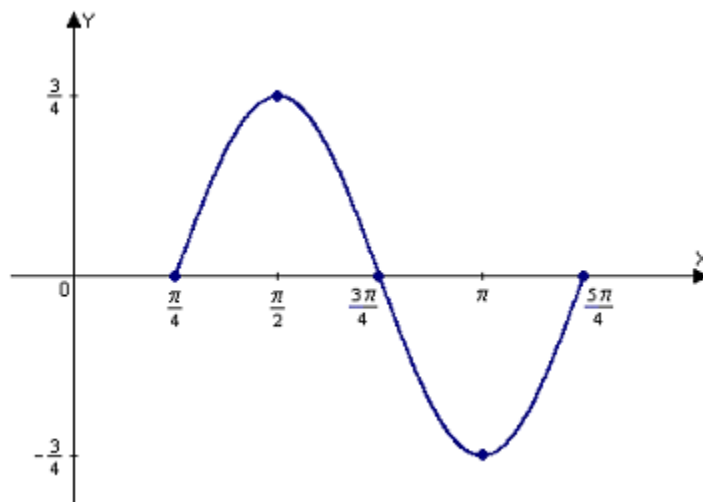
Find the y-values of the key points of the period

x-value	y-value	key point
$\frac{\pi}{4}$	$y = \frac{3}{4} \sin\left(2\left(\frac{\pi}{4}\right) - \frac{\pi}{2}\right)$ $= \frac{3}{4} \sin(0)$ $= \frac{3}{4}(0)$ $= 0$	$\left(\frac{\pi}{4}, 0\right)$
$\frac{\pi}{2}$	$y = \frac{3}{4} \sin\left(2\left(\frac{\pi}{2}\right) - \frac{\pi}{2}\right)$ $= \frac{3}{4} \sin\left(\frac{\pi}{2}\right)$ $= \frac{3}{4}(1)$ $= \frac{3}{4}$	$\left(\frac{\pi}{2}, \frac{3}{4}\right)$
$\frac{3\pi}{4}$	$y = \frac{3}{4} \sin\left(2\left(\frac{3\pi}{4}\right) - \frac{\pi}{2}\right)$ $= \frac{3}{4} \sin(\pi)$ $= \frac{3}{4}(0)$ $= 0$	$\left(\frac{3\pi}{4}, 0\right)$
π	$y = \frac{3}{4} \sin\left(2(\pi) - \frac{\pi}{2}\right)$ $= \frac{3}{4} \sin\left(\frac{3\pi}{2}\right)$ $= \frac{3}{4}(-1)$ $= -\frac{3}{4}$	$\left(\pi, -\frac{3}{4}\right)$

Exercise 3 (Continued):

$\frac{5\pi}{4}$	$y = \frac{3}{4} \sin\left(2\left(\frac{5\pi}{4}\right) - \frac{\pi}{2}\right)$ $= \frac{3}{4} \sin(2\pi)$ $= \frac{3}{4}(0)$ $= 0$	$\left(\frac{5\pi}{4}, 0\right)$
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Graph the function



Exercise 4: Graph the given trigonometric function over one complete period.

$$y = -\sin\left(x + \frac{2\pi}{3}\right)$$

Find the amplitude

$$\begin{aligned} \text{amplitude} &= |A| \\ &= |-1| \\ &= 1 \end{aligned}$$

Exercise 4 (Continued):

Find the period

$$\begin{aligned} \text{period} &= \frac{2\pi}{B} \\ &= \frac{2\pi}{1} \\ &= 2\pi \end{aligned}$$

Find the phase shift

$$\begin{aligned} y &= -\sin\left(x + \frac{2\pi}{3}\right) \\ y &= -\sin\left(x - \left(-\frac{2\pi}{3}\right)\right) \\ \text{phase shift} &= \frac{C}{B} \\ &= \frac{-\frac{2\pi}{3}}{1} \\ &= -\frac{2\pi}{3} \end{aligned}$$

Find the x-values of the key points of the period

$$\begin{aligned} \text{interval width} &= \frac{\text{period}}{4} \\ &= \frac{2\pi}{4} \\ &= \frac{\pi}{2} \end{aligned}$$

The first key point will start at $x = -\frac{2\pi}{3}$ and increase by $\frac{\pi}{2}$

$$x_1 = -\frac{2\pi}{3} \quad x_2 = -\frac{2\pi}{3} + \frac{\pi}{2} \quad x_3 = -\frac{\pi}{6} + \frac{\pi}{2} \quad x_4 = \frac{\pi}{3} + \frac{\pi}{2} \quad x_5 = \frac{5\pi}{6} + \frac{\pi}{2}$$

$$x_1 = -\frac{2\pi}{3} \quad x_2 = -\frac{\pi}{6} \quad x_3 = \frac{\pi}{3} \quad x_4 = \frac{5\pi}{6} \quad x_5 = \frac{4\pi}{3}$$

Exercise 4 (Continued):

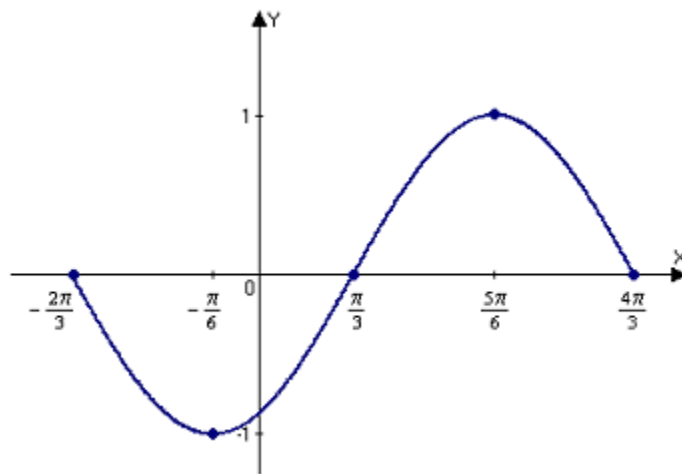
Find the y-values of the key points of the period

x-value	y-value	key point
$-\frac{2\pi}{3}$	$y = -\sin\left(x + \frac{2\pi}{3}\right)$ $= -\sin\left(-\frac{2\pi}{3} + \frac{2\pi}{3}\right)$ $= -\sin(0)$ $= 0$	$\left(-\frac{2\pi}{3}, 0\right)$
$-\frac{\pi}{6}$	$y = -\sin\left(x + \frac{2\pi}{3}\right)$ $= -\sin\left(-\frac{\pi}{6} + \frac{2\pi}{3}\right)$ $= -\sin\left(\frac{\pi}{2}\right)$ $= -1$	$\left(-\frac{\pi}{6}, -1\right)$
$\frac{\pi}{3}$	$y = -\sin\left(x + \frac{2\pi}{3}\right)$ $= -\sin\left(\frac{\pi}{3} + \frac{2\pi}{3}\right)$ $= -\sin(\pi)$ $= 0$	$\left(\frac{\pi}{3}, 0\right)$
$\frac{5\pi}{6}$	$y = -\sin\left(x + \frac{2\pi}{3}\right)$ $= -\sin\left(\frac{5\pi}{6} + \frac{2\pi}{3}\right)$ $= -\sin\left(\frac{3\pi}{2}\right)$ $= 1$	$\left(\frac{5\pi}{6}, 1\right)$

Exercise 4 (Continued):

x-value	y-value	key point
$\frac{4\pi}{3}$	$y = -\sin\left(x + \frac{2\pi}{3}\right)$ $= -\sin\left(\frac{4\pi}{3} + \frac{2\pi}{3}\right)$ $= -\sin(2\pi)$ $= 0$	$\left(\frac{4\pi}{3}, 0\right)$

Graph the function



Exercise 5: Graph the given trigonometric function over one complete period.

$$y = 2 \cos(2\pi x)$$

Find the amplitude

$$\begin{aligned} \text{amplitude} &= |A| \\ &= |2| \\ &= 2 \end{aligned}$$

Find the period

$$\begin{aligned} \text{period} &= \frac{2\pi}{B} \\ &= \frac{2\pi}{2\pi} \\ &= 1 \end{aligned}$$

Exercise 5 (Continued):

Find the x-values of the key points of the period

$$\begin{aligned} \text{interval width} &= \frac{\text{period}}{4} \\ &= \frac{1}{4} \end{aligned}$$

The first key point will start at $x = 0$ and increase by $\frac{1}{4}$

$$x_1 = 0 \quad x_2 = 0 + \frac{1}{4} \quad x_3 = \frac{1}{4} + \frac{1}{4} \quad x_4 = \frac{1}{2} + \frac{1}{4} \quad x_5 = \frac{3}{4} + \frac{1}{4}$$

$$x_1 = 0 \quad x_2 = \frac{1}{4} \quad x_3 = \frac{1}{2} \quad x_4 = \frac{3}{4} \quad x_5 = 1$$

Find the y-values of the key points of the period

x-value	y-value	key point
0	$y = 2 \cos(2\pi(0))$ $= 2 \cos(0)$ $= 2$	(0, 2)
$\frac{1}{4}$	$y = 2 \cos\left(2\pi\left(\frac{1}{4}\right)\right)$ $= 2 \cos\left(\frac{\pi}{2}\right)$ $= 0$	$\left(\frac{1}{4}, 0\right)$
$\frac{1}{2}$	$y = 2 \cos\left(2\pi\left(\frac{1}{2}\right)\right)$ $= 2 \cos(\pi)$ $= -2$	$\left(\frac{1}{2}, -2\right)$
$\frac{3}{4}$	$y = 2 \cos\left(2\pi\left(\frac{3}{4}\right)\right)$ $= 2 \cos\left(\frac{3\pi}{2}\right)$ $= 0$	$\left(\frac{3}{4}, 0\right)$

Exercise 5 (Continued):

x-value	y-value	key point
1	$y = 2 \cos(2\pi(1))$ $= 2 \cos(2\pi)$ $= 2$	(1, 2)

Graph the function

