Review Exercise Set 18

Exercise 1: Betty needs to grow a culture of 150,000 bacteria, which are known to double in number every 12 hours. If Betty starts off with 1,000 bacteria, find the growth rate and how many hours it will take for the culture to reach the desired amount.

Exercise 2: If the half-life of carbon-14, a radioactive compound found in all living organisms, is 5,700 years, estimate the age of a mummy found having only 20% of the normal amount of carbon-14.

Exercise 3: The number of people in a city that got the flu t weeks after its initial outbreak is modeled with the following function.

$$f(t) = \frac{400000}{1 + 2300e^{-0.93t}}$$

Find how many people were sick when the flu began and how many people were sick after 3 weeks.

Exercise 4: You get a cup of coffee from the coffee pot in your office and it has an initial temperature of 175° F. The room temperature in your office is a constant 78° F. After 5 minutes, the temperature of the coffee has dropped to 155° F. Write a model for the temperature of the coffee after t minutes.

Exercise 5: Using the model derived in Exercise 4, determine how long it will take for the temperature of the coffee to drop to 90° F.

Review Exercise Set 18 Answer Key

Exercise 1: Betty needs to grow a culture of 150,000 bacteria, which are known to double in number every 12 hours. If Betty starts off with 1,000 bacteria, find the growth rate and how many hours it will take for the culture to reach the desired amount.

Find the growth rate

$A = A_0\, e^{kt}$
$2A_0 = A_0\, e^{k(12)}$
$2 = e^{12k}$
$\ln 2 = 12k$
$\frac{1}{12} \ln 2 = k$
$0.058 \approx k$

Find the number of hours for the culture to grow in size to 150,000

$A = A_0\, e^{kt}$
$150000 = (1000)\, e^{0.058t}$
$150 = e^{0.058t}$
$\ln 150 = 0.058t$
$\frac{\ln 150}{0.058} = t$
$86.75 \approx t$

It will take approximately $86\frac{3}{4}$ hours

Exercise 2: If the half-life of carbon-14, a radioactive compound found in all living organisms, is 5,715 years, estimate the age of a mummy found having only 20% of the normal amount of carbon-14.

Find the decay rate

$A = A_0\, e^{kt}$
$\frac{1}{2} A_0 = A_0\, e^{k(5715)}$
$\frac{1}{2} = e^{5715k}$
$\ln \frac{1}{2} = 5715k$
$\ln 2^{-1} = 5715k$
$-\ln 2 = 5715k$
$\frac{-\ln 2}{5715} = k$
$-0.0001213 \approx k$

Exercise 2 (Continued):

Estimate the age of the mummy

$A = A_0\, e^{kt}$
$.2A_0 = A_0\, e^{-0.0001213t}$

$.2 = e^{-0.0001213t}$
$\ln .2 = -0.0001213t$
$\frac{\ln .2}{-0.0001213} = t$
$13268 \approx t$

The mummy is approximately 13,268 years old.

Exercise 3: The number of people in a city that got the flu t weeks after its initial outbreak is modeled with the following function.

$$f(t) = \frac{400,000}{1 + 2300e^{-0.93t}}$$

Find how many people were sick when the flu began and how many people were sick after 3 weeks.

Find the number of people sick at the beginning of the outbreak (t = 0)

$$\begin{aligned} f(0) &= \frac{400,000}{1 + 2300e^{-0.93(0)}} \\ &= \frac{400,000}{1 + 2300e^{0}} \\ &= \frac{400,000}{1 + 2300} \\ &\approx 173.837 \end{aligned}$$

There were approximately 174 people sick when the flu began

Exercise 3 (Continued):

Find the number of people sick after 3 weeks (t = 3)

$$
\begin{aligned}
f(3) &= \frac{400,000}{1+2300e^{-0.93(3)}} \\
&= \frac{400,000}{1+2300e^{-2.79}} \\
&\approx \frac{400,000}{1+141.2688} \\
&\approx 2811.579
\end{aligned}
$$

There were approximately 2,812 people sick after 3 weeks

Exercise 4: You get a cup of coffee from the coffee pot in your office and it has an initial temperature of 175° F. The room temperature in your office is a constant 78° F. After 5 minutes, the temperature of the coffee has dropped to 155° F. Write a model for the temperature of the coffee after t minutes.

Use Newton's Law of Cooling

$T_0 = 175$; $C = 78$

$T = C + (T_0 - C)e^{kt}$
$T = 78 + (175 - 78)e^{kt}$
$T = 78 + 97e^{kt}$

Determine the value of k using the information that the temperature is 155° F after 5 minutes.

$T = 155$; $t = 5$

$T = 78 + 97e^{kt}$
$155 = 78 + 97e^{k(5)}$
$77 = 97e^{5k}$
$\frac{77}{97} = e^{5k}$
$\ln \frac{77}{97} = 5k$
$\frac{1}{5} \ln \frac{77}{97} = k$
$-0.0462 \approx k$

Exercise 4 (Continued):

Substitute the value of k into the model to get the function

$$T = 78 + 97e^{-0.0462t}$$

Exercise 5: Using the model derived in Exercise 4, determine how long it will take for the temperature of the coffee to drop to 90° F.

Substitute 90 for T and solve for t

$$T = 78 + 97e^{-0.0462t}$$
$$90 = 78 + 97e^{-0.0462t}$$
$$12 = 97e^{-0.0462t}$$
$$\frac{12}{97} = e^{-0.0462t}$$
$$\ln\frac{12}{97} = -0.0462t$$
$$\frac{\ln\frac{12}{97}}{-0.0462} = t$$
$$45.234 \approx t$$

It will take approximately 45 minutes 14 seconds for the temperature to drop to 90° F.