Basic functions and Their Graphs

For many situations in life, we can see that one quantity depends upon another. For example, if you are traveling in a car, the distance you travel depends on the time you have been traveling. The term *function* is used to describe these types of relationships. We define the term function in the following way:

A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

Functions can be represented in several ways:
    (1) Verbally
    (2) Algebraically
    (3) Numerically
    (4) Visually

We will now consider each of the above representations by looking at the relationship between the circumference of a circle and its diameter.
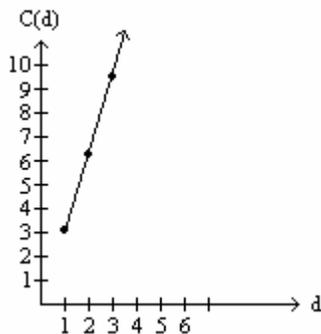
    (1) Verbally, we would say that the circumference of a circle is a function of its diameter.

    (2) Algebraically, we could represent the circumference of a circle in the following way:

$$C(d) = \Pi d$$

where d is the diameter and $C(d)$ is the circumference (a function of the diameter).

    (3) Numerically, we can create a table of values:

d	C(d)
1	$\pi \approx 3.14$
2	$2\pi \approx 6.28$
3	$3\pi \approx 9.42$

    (4) Visually, we can graph the above values:

It is important to review some terminology before continuing. In the definition of a function, two sets *A* and *B* were mentioned. Set *A* is called the **domain** of the function, and set *B* is called the **range** of the function. The domain of any function consists of the independent variables (for example, the *d* values in the example above). The range consists of the dependent variables. In our example, these would be the *C(d)* values, because the circumference *depends* on the diameter. (We will speak more about the domain shortly.)
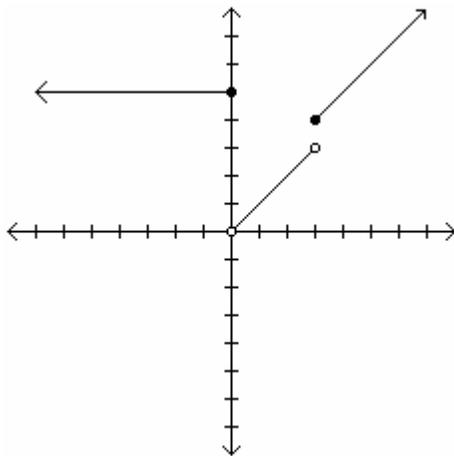
The Vertical Line Test:

The vertical line test is a very simple way to determine if a graph is the graph of a function or not. You have all seen the graph of a circle, but is this the graph of a function? No, as can be easily seen by the *vertical line test.*

Def: The Vertical Line Test:
A curve in the coordinate plane is the graph of a function if and only if no vertical line intersects the curve more than once. Simply stated, if you can draw a vertical line that touches the curve at more than one point, the curve is not the graph of a function-- even if there is only one vertical line for which this is true.

Let's consider the following graph as an example.

At first, it may seem that this is not a function according to the vertical line test. If you draw a vertical line at $x = 0$ or $x = 3$, will it touch the graph at more than one point? Remember that the open dot means that the point is *not* included in the graph; therefore, the vertical line would only intersect the graph at one point in these two instances. We see that it does pass the vertical line test, and so it is indeed the graph of a function.

With a circle, it is easy to see that multiple vertical lines can be drawn that would touch the graph at two different points. Therefore, a circle is *not* the graph of a function.

Evaluating functions:

Evaluating a function simply means that we find the value of the function at a particular value in the domain. This is accomplished by substituting the number in for the independent variable.

Example 1:

Let $f(x) = x^3 - 3x^2 + 5$ Evaluate the function at $x = -2$.

In this case, x is the independent variable, so we will substitute the value -2 for x.

$$f(-2) = (-2)^3 - 3(-2)^2 + 5$$

$$= (-8) - 3(4) + 5$$

$$= -15$$

Determining the domain of a function:

Unless the domain of a function is defined explicitly (i.e. $0<x<5$), then the domain is the set of all values of x for which the function is defined as a real number.

Two things to remember:

(1) We can never take the square root of a negative number if we want to get a real number

(2) We can never divide by 0

Example 2:

Find the domain of the following function:

$f(x) = \sqrt{x^2 - 4}$ (Remember, we do not take the square root of a negative number if we want to get a real number.)

$x^2 - 4 \geq 0$ We set the radicand greater than or equal to zero and factor the inequality.

$x \leq -2$ or $x \geq 2$ Using methods from previous courses, we can solve the inequality to find that $x \leq -2$ or $x \geq 2$.

So, in interval notation, the domain is $(-\infty, -2] \cup [2, \infty)$

Example 3:

Find the domain of the following function:

$$f(x) = \frac{1}{2x-6}$$

(Remember, the function cannot be defined when the denominator is 0, so we must determine where this occurs.)

$$2x - 6 = 0$$
$$2x = 6$$
$$x = 3$$

We set the denominator equal to zero in order to see where the function would be undefined.

Thus, we see that when x=3, the function is undefined. The domain is all real numbers *except* 3.

In set-builder notation, this would be written as $\{x \mid x \neq 3\}$