Modeling Functions

In earlier sections, we saw how we can write formulas, or equations, in order to represent real-life situations. We call this *mathematical modeling*. We can use mathematical modeling to solve problems in many areas such as science, business, and medicine.

Before we can set up an equation as a mathematical model, we must determine what the variable will be and organize all of the information in terms of that variable. For example, let's assume that all of the SLAC lab employees will receive a $150 Christmas bonus. If the college must pay $3,300 to cover the cost of the Christmas bonuses, how many employees must be working in the SLAC lab?

We want to know how many employees work in the SLAC lab, so we will set e to be our variable; therefore, we know that e represents the number of employees in the lab. Now, we organize the other information in terms of the variable e. If each employee receives $150, then the total amount of money paid will be $\$150e$. (The information given in this problem is simple, but in some cases it will be helpful to set up a table in order to organize the information.)

Now, we can set up the model and solve for the unknown variable.

$$\$3{,}300 = \$150e$$
$$e = 22 \qquad \text{(Divide by \$150.)}$$

There are 22 employees in the SLAC lab. We can check our answer by substituting 22 in for e.

We can summarize the steps we used by the following guidelines.

Modeling with Equations:

1. *Determine the variable*. The desired quantity will usually be found in the last sentence of the problem. The notation for this value (*x,a,c,etc.*) can be anything you choose to call it, but it is helpful if it, in some way, represents the desired quantity.

2. *Express all unknown quantities in terms of the variable.* In some cases, it may be helpful to make a table with the information.

3. *Set up the model.* In equation form, organize the expressions listed in Step 2.

4. *Solve the equation and check your answer.* Be sure to express the answer in sentence form, clearly stating the answer to the question in the problem.

Now, let's look at some more challenging examples.

Example 1:

Carolyn is planning to visit the dentist. When she calls to make the appointment, she finds out that the initial check-up is \$65, and she can have her teeth cleaned for an additional \$80. For each cavity that is found, she must pay an additional \$115 to have it filled. Assuming that she decides to have her teeth cleaned, how many cavities must she have if her total bill is \$605?

We will choose c to represent the number of cavities. Setting up a table with the additional information, we have:

In words:	In algebra:
Number of cavities	c
Amount spent on filling the cavities	\$115c
Total amount spent at the dentist	\$65 + \$80 + \$115c

We know the total amount spent at the dentist is \$605, we can set up our equation (model).

$$605 = 65 + 80 + 115c$$

Now, we can solve for c, which will give us the number of cavities.

$$605 = 65 + 80 + 115c$$
$$605 = 145 + 115c$$
$$460 = 115c$$
$$\frac{460}{115} = c$$
$$c = 4$$

Carolyn had four cavities filled.

Example 2:

Patrick and Tucker decide to have a bicycle race to the store. Since Patrick has recently had knee surgery, they decide that he should have a 15 minute head start. If Patrick rides at a speed of 12 mi/h, and Tucker rides at a speed of 16 mi/h, how long will it take Tucker to catch up with him?

We are looking for the amount of time it took Tucker to catch up with Patrick. We use t to represent this quantity. Before we can set up a table, we must recall an important formula:

$$\text{distance} = \text{rate} \times \text{time}$$

We will express all distances in terms of miles and all times in terms of hours, in order to stay consistent. Keeping times in terms of hours, we see that Patrick's head start is $\frac{1}{4}$ hour.

In words:	In algebra:
Time Tucker rode	t
Time Patrick rode	$t+\frac{1}{4}$
Rate at which Tucker rode	16 mi/h
Rate at which Patrick rode	12 mi/h
Distance Tucker rode	$16t$
Distance Patrick rode	$12\left(t+\frac{1}{4}\right)$

We can also set this information up in another type of table, which may be easier. (To understand this table, remember that the distance traveled is equal to the rate multiplied by the time.)

	Distance	Rate	Time
Tucker	$16t$	16	t
Patrick	$12(t+1/4)$	12	$t+1/4$

We want to know how long it took for Tucker to catch up with Patrick. At the moment that he catches up with Patrick, their distance cycled should be the same, so we set up the following equation:

$$16t = 12(t+\frac{1}{4})$$

$$16t = 12t+3$$ Using the distributive property.

$$4t = 3$$ Combining like terms.

$$t = \frac{3}{4}$$ Dividing by 4.

Since the time is in hours, we see that it takes 45 minutes for Tucker to catch up with Patrick.

Example 3:

The manager of Common Grounds coffee shop has noticed that many of the college-aged customers prefer coffee that is less than $2.00 per pound. In order to create more options, he decides to blend a certain coffee that sells for $1.80 per pound with a coffee that sells for $2.00 per pound. He is hoping to make 40 pounds of the new blend, which will cost $1.92 per pound. How much of each type of coffee must he blend in order to get the desired amount of coffee?

We are looking for two different amounts in this problem: the amount of $1.80 per pound coffee, and the amount of $2.00 per pound coffee. We will let *c* represent the amount of $1.80 per pound coffee, and then we will write the other amount in terms of *c*.

In words:	In algebra:
Amount of $1.80 coffee	c
Amount of $2.00 coffee	$40 - c$ (We know the total amount is 40 Pounds.)

*Note: Instead of writing $1.80 and $2.00 throughout the problem, we will write 1.8 and 2 in order to simplify the equation.

We can put this information together in an equation:

$$1.8c + 2(40 - c) = 1.92(40)$$
$$1.8c + 80 - 2c = 76.8$$
$$-0.2c + 80 = 76.8$$
$$-0.2c = -3.2$$
$$c = 16$$

So, he will need 16 pounds of the $1.80 per pound coffee. He will need (40-16) or 24 pounds of the coffee that costs $2.00 per pound.