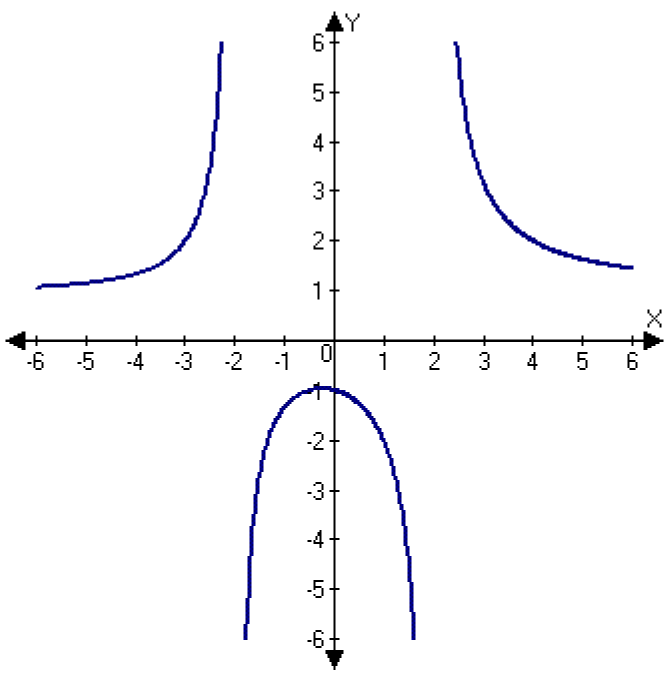
Review Exercise Set 12

Exercise 1: Find the domain of the given rational function.

$$h(x) = \frac{x-5}{x^2+4x+3}$$

Exercise 2: Use the given graph to complete the statements below.



a) $As\ x \to -\infty,\ f(x) \to$

b) $As\ x \to -2^-,\ f(x) \to$

c) $As\ x \to -2^+,\ f(x) \to$

d) $As\ x \to 2^-,\ f(x) \to$

e) $As\ x \to 2^+,\ f(x) \to$

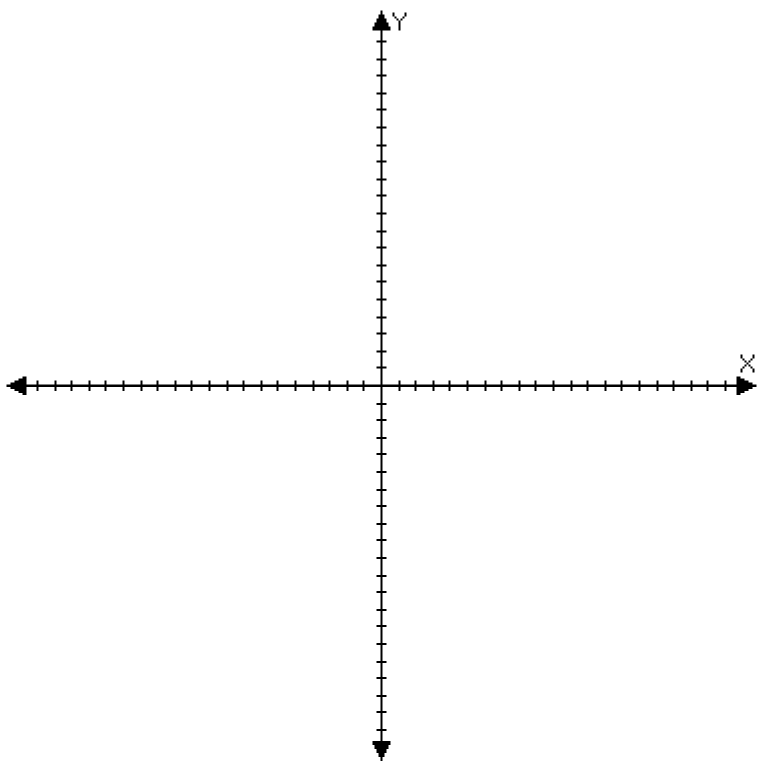
f) $As\ x \to \infty,\ f(x) \to$

Exercise 3: Find the vertical and horizontal asymptotes of the given rational function.

$$g(x) = \frac{3x+7}{x^2 - x - 6}$$

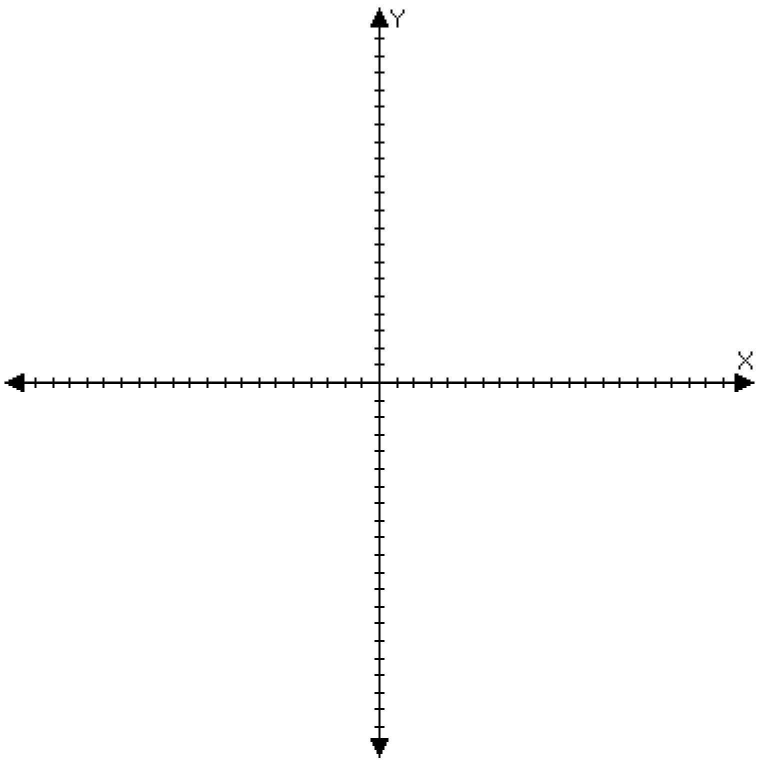
Exercise 4: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

$$r(x) = \frac{2x^2}{x^2 + 4x - 12}$$



Exercise 5: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

$$r(x) = \frac{x^3 + 2}{x^2 + x}$$



Review Exercise Set 12 Answer Key

Exercise 1: Find the domain of the given rational function.

$$h(x) = \frac{x-5}{x^2+4x+3}$$

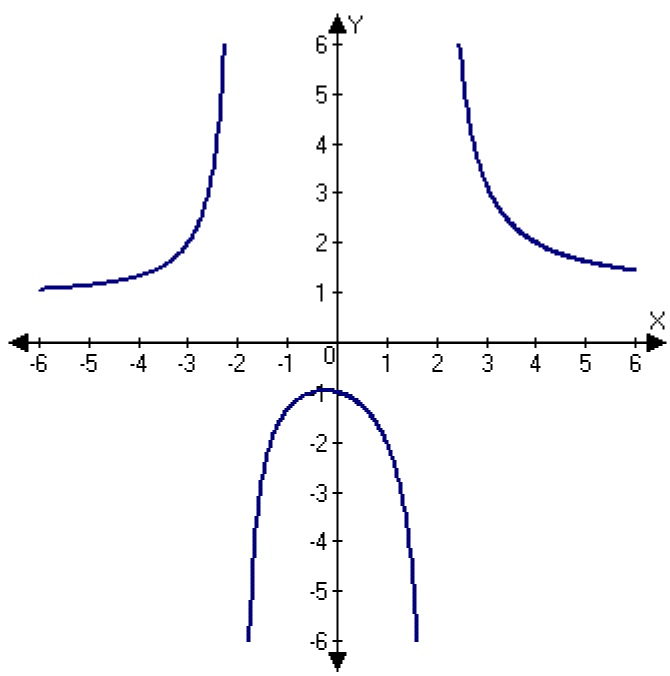
Set the denominator equal to zero and solve for x

$x^2 + 4x + 3 = 0$
$(x + 1)(x + 3) = 0$
$x + 1 = 0$ or $x + 3 = 0$
$x = -1$ or $x = -3$

Exclude the values that make the denominator zero from the domain

Domain: $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$

Exercise 2: Use the given graph to complete the statements below.



a) $As\ x \to -\infty,\ f(x) \to$

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e) $As\ x \to 2^+,\ f(x) \to$

f) $As\ x \to \infty,\ f(x) \to$

Exercise 3: Find the vertical and horizontal asymptotes of the given rational function.

$$g(x) = \frac{3x+7}{x^2 - x - 6}$$

Vertical asymptote

Set the denominator equal to zero and solve for x

$x^2 - x - 6 = 0$
$(x + 2)(x - 3) = 0$
$x + 2 = 0$ or $x - 3 = 0$
$x = -2$ or $x = 3$

The vertical asymptotes will be at $x = -2$ and $x = 3$

Horizontal asymptote

Compare degrees of the numerator and denominator

degree of numerator: 1
degree of denominator: 2

Since the denominator has a larger degree, the horizontal asymptote will be the x-axis or $y = 0$.

Exercise 4: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

$$r(x) = \frac{2x^2}{x^2 + 4x - 12}$$

Vertical asymptote

$x^2 + 4x - 12 = 0$
$(x + 6)(x - 2) = 0$
$x + 6 = 0$ or $x - 2 = 0$
$x = -6$ or $x = 2$

Horizontal asymptote

degree of numerator: 2
degree of denominator: 2

degrees are the same so the horizontal asymptote will be the ratio of the leading coefficients.

$y = \frac{2}{1} = 2$

Exercise 4 (Continued):

Intercepts

Let x = 0

$$r(0)=\frac{2(0)^2}{(0)^2+4(0)-12}$$
$$r(0)=0$$

y-intercept (0, 0)

Let r(x) = 0

$$0=\frac{2x^2}{x^2+4x-12}$$
$$0=2x^2$$
$$0=x^2$$
$$0=x$$

x-intercept (0, 0)

Symmetry

$$r(-x)=r(x)$$
$$\frac{2(-x)^2}{(-x)^2+4(-x)-12}=\frac{2x^2}{x^2+4x-12}$$
$$\frac{2x^2}{x^2-4x-12}=\frac{2x^2}{x^2+4x-12}$$

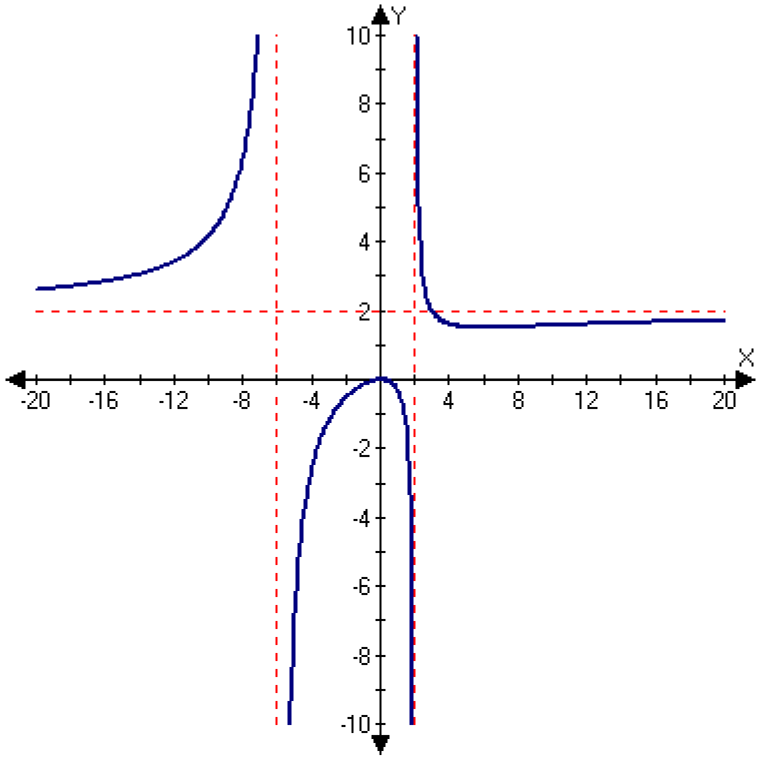
r(-x) and r(x) are not the same functions so it is not symmetric about the y-axis.

$$r(-x)=-r(x)$$
$$\frac{2(-x)^2}{(-x)^2+4(-x)-12}=-\frac{2x^2}{x^2+4x-12}$$
$$\frac{2x^2}{x^2-4x-12}=\frac{-2x^2}{x^2+4x-12}$$

r(-x) and -r(x) are not the same functions so it is not symmetric about the origin.

Exercise 4 (Continued):

Graph



Exercise 5: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

$$r(x) = \frac{x^3 + 2}{x^2 + x}$$

Vertical asymptote

$x^2 + x = 0$
$x(x + 1) = 0$
$x = 0$ or $x + 1 = 0$
$x = 0$ or $x = -1$

Horizontal asymptote

degree of numerator: 3
degree of denominator: 2

The numerator has the larger degree so there is no horizontal asymptote. However, since the difference in the degrees is 1 there will be a slant asymptote.

Exercise 5 (Continued):

Slant asymptote

Divide the rational function using long division

$$
\begin{array}{r}
x - 1 \\
x^2 + x \overline{\big) x^3 + 0x^2 + 0x + 2} \\
\underline{x^3 + x^2} \qquad\quad \\
-x^2 + 0x \qquad \\
\underline{-x^2 - x} \qquad \\
x + 2
\end{array}
$$

The quotient of x - 1 is the slant asymptote.

Intercepts

Let x = 0

x cannot be zero since this is the location of one of the vertical asymptotes

Let r(x) = 0

$$0 = \frac{x^3 + 2}{x^2 + x}$$

$$0 = x^3 + 2$$

$$-2 = x^3$$

$$-\sqrt[3]{2} = x$$

$$-1.26 \approx x$$

x-intercept $\left(-\sqrt[3]{2}, 0\right)$

Symmetry

$$r(-x) = r(x)$$

$$\frac{(-x)^3 + 2}{(-x)^2 + (-x)} = \frac{x^3 + 2}{x^2 + x}$$

$$\frac{-x^3 + 2}{x^2 - x} = \frac{x^3 + 2}{x^2 + x}$$

r(-x) and r(x) are not the same functions so it is not symmetric about the y-axis.

Exercise 5 (Continued):

$$r(-x) = -r(x)$$
$$\frac{(-x)^3 + 2}{(-x)^2 + (-x)} = -\frac{x^3 + 2}{x^2 + x}$$
$$\frac{-x^3 + 2}{x^2 - x} = \frac{-x^3 - 2}{x^2 + x}$$

r(-x) and -r(x) are not the same functions so it is not symmetric about the origin.

Graph