Review Exercise Set 27

Exercise 1: Evaluate the given binomial coefficient.

$$\binom{15}{7}$$

Exercise 2: Expand the given expression using the binomial theorem.

$(x + 2)^3$

Exercise 3: Expand the given expression using the binomial theorem.

$(3x - 4y)^5$

Exercise 4: Find the indicated term of the binomial expansion for the given expression.

5^{th} term of $(2x - 5y)^8$

Exercise 5: Find the indicated term of the binomial expansion for the given expression.

8^{th} term of $(x + 3y)^{10}$

Review Exercise Set 27 Answer Key

Exercise 1: Evaluate the given binomial coefficient.

$$\binom{15}{7}$$

$$\begin{aligned}\binom{15}{7} &= \frac{15!}{7!(15-7)!} \\ &= \frac{15!}{7!8!} \\ &= \frac{15\times14\times13\times12\times11\times10\times9\times8!}{7\times6\times5\times4\times3\times2\times1\times8!} \\ &= 13\times11\times5\times9 \\ &= 6,435\end{aligned}$$

Exercise 2: Expand the given expression using the binomial theorem.

$(x + 2)^3$

$$= \binom{3}{0}(x)^3 + \binom{3}{1}(x)^2(2)^1 + \binom{3}{2}(x)^1(2)^2 + \binom{3}{3}(2)^3$$

$$== \frac{3!}{0!3!}\left(x^3\right) + \frac{3!}{1!2!}\left(2x^2\right) + \frac{3!}{2!1!}(4x) + \frac{3!}{3!0!}(8)$$

$$= (1)\left(x^3\right) + (3)\left(2x^2\right) + (3)(4x) + (1)(8)$$

$$= x^3 + 6x^2 + 12x + 8$$

Exercise 3: Expand the given expression using the binomial theorem.

(3x - 4y)^5

$$= \binom{5}{0}(3x)^5 + \binom{5}{1}(3x)^4(-4y)^1 + \binom{5}{2}(3x)^3(-4y)^2 + \binom{5}{3}(3x)^2(-4y)^3$$
$$+ \binom{5}{4}(3x)^1(-4y)^4 + \binom{5}{5}(-4y)^5$$

$$= \frac{5!}{0!5!}(243x^5) + \frac{5!}{1!4!}(81x^4)(-4y) + \frac{5!}{2!3!}(27x^3)(16y^2) + \frac{5!}{3!2!}(9x^2)(-64y^3)$$
$$+ \frac{5!}{4!1!}(3x)(256y^4) + \frac{5!}{5!0!}(-1024y^5)$$

= (1)(243x^5) + (5)(-324x^{4}y) + (10)(432x^{3}y^2) + (10)(-576x^{2}y^3) + (5)(768xy^4) + (1)(-1024y^5)
= 243x^5 - 1620x^{4}y + 4320x^{3}y^2 - 5760x^{2}y^3 + 3840xy^4 - 1024y^5

Exercise 4: Find the indicated term of the binomial expansion for the given expression.

5th term of (2x - 5y)^8

Find the value of r

r is always 1 more than the term so for the 5th term

r + 1 = 5
r = 4

Find the 5th term

n = 8; r = 4; a = 2x; b = -5y

$$\binom{n}{r} a^{n-r} b^r$$
$$= \binom{8}{4}(2x)^{8-4}(-5y)^4$$
$$= \frac{8!}{4!4!}(16x^4)(625y^4)$$
$$= (70)(10{,}000x^4y^4)$$
$$= 700{,}000x^4y^4$$

Exercise 5: Find the indicated term of the binomial expansion for the given expression.

8^{th} term of $(x + 3y)^{10}$

Find the value of r

For the 8^{th} term

$r + 1 = 8$
$r = 7$

Find the 8^{th} term

$n = 10$; $r = 7$; $a = x$; $b = 3y$

$$\binom{n}{r} a^{n-r} b^r$$

$$= \binom{10}{7} (x)^{10-7} (3y)^7$$

$$= \frac{10!}{7!3!} (x^3)(2187y^7)$$

$$= (120)(2187x^3y^7)$$

$$= 262,440x^3y^7$$