

Scientific Notation

In the sciences, many of the things measured or calculated involve numbers that are either very large or very small. As a result, it is inconvenient to write out such long numbers or to perform calculations long hand. In addition, most calculators do not have enough window space to be able to show these long numbers. To remedy this, we have available a short hand method of representing numbers called scientific notation. The chart below gives you some examples of powers of 10 and their names and equivalences.

Exponent	Expanded	Prefix	Symbol	Name	Fraction
10^{-12}	0.000000000001	pico-	p	one trillionth	1/1,000,000,000,000
10^{-9}	0.000000001	nano-	n	one billionth	1/1,000,000,000
10^{-6}	0.000001	micro-	u	one millionth	1/1,000,000
10^{-3}	0.001	milli-	m	one thousandth	1/1,000
10^{-2}	0.01	centi-	c	one hundredth	1/100
10^{-1}	0.1	deci-	d	one tenth	1/10
10^0	1	-----	-----	one	-----
10^1	10	Deca-	D	ten	-----
10^2	100	Hecto-	H	hundred	-----
10^3	1,000	Kilo-	k	thousand	-----
10^4	10,000	-----	10k	ten thousand	-----
10^5	100,000	-----	100k	one hundred thousand	-----
10^6	1,000,000	Mega-	M	one million	-----
10^9	1,000,000,000	Giga-	G	one billion	-----
10^{12}	1,000,000,000,000	Tera-	T	one trillion	-----
10^{15}	1,000,000,000,000,000	Peta-	P	one quadrillion	-----

Powers of 10 and Place Value.....

Multiplying by 10, 100, or 1000 in the following problems just means to add the number of zeroes to the number being multiplied. This is because our number system is based on 10. The chart above shows the powers of 10 you are most likely to encounter in your science studies.

1. $35 \times 10 \rightarrow 35 + 0 \rightarrow 350$
2. $6 \times 100 \rightarrow 6 + 0 + 0 \rightarrow 600$
3. $925 \times 10 \rightarrow 925 + 0 \rightarrow 9,250$
4. $42 \times 1000 \rightarrow 42 + 0 + 0 + 0 \rightarrow 42,000$
5. $691 \times 1000 \rightarrow 691 + 0 + 0 + 0 \rightarrow 691,000$

Places to right of the decimal point are called *decimal fractions*. The negative exponents shown under the negative exponents shown under the Exponents column above tell you to divide by that number.

Examples:

$$10^{-1} = 1/10 = .1$$
$$10^{-2} = 1/10^2 = 1/100 = .01$$
$$10^{-3} = 1/10^3 = 1/1000 = .001$$

You know that the value of each digit depends on which place it occupies. For example, the 5 in 5628 has the value of 5 thousand, while the value of the 1 in 1586000 is 1 million. A number can be expanded according to the place value it holds:

Example A:

$$589 = 5(100) + 8(10) + 9(1)$$
$$67.32 = 6(10) + 7(1) + 3(.1) + 2(.01)$$

For larger numbers, it is easier to use exponents for the place values.

Example B:

$$96,734,000 = 9(10^7) + 6(10^6) + 7(10^5) + 3(10^4) + 4(10^3)$$

When you expand the numbers you should notice these two facts:

1. When a power of 10 is written the long way, the number of zeroes behind the 1 is the same as the exponent of the 10 when it is written in shorthand.

Examples:

$$100 = 10^2 ; (2 \text{ zeroes so exponent is } 2)$$
$$100,000 = 10^5 ; (5 \text{ zeroes so exponent is } 5)$$

2. The number of places between the digit and the decimal point is the same as the exponent of the 10 in the place value of that digit. Consider the two following examples:
 - a. In the number 5,078.4 look at the 5. There are 3 places between it and the decimal point. Its place value is 10^3 .
 - b. In the number 56,700,000.6 look at the 5. There are 7 places between it and the decimal point. Its place value is 10^7 .

On the place value chart below, the place value names do *not* center around the decimal point. Instead they center around the *ones place*. The names to the left of the ones place match up with the names to the right.

thousands.....thousandths
hundreds.....hundredths
tens.....tenths
ones

Example Set 1.

What is the value of 8 in each of the following numbers?

- a) 84.67 _____
- b) 209.82 _____
- c) 38,009 _____
- d) 85,000,000 _____
- e) 0.08 _____

Example Set 2.

Expand each of the following numbers by place (See examples above)

a) 380 _____

b) 5000.02 _____

c) 60,400 _____

d) 29,000,000 _____

e) 100.004 _____

Example Set 3.

Write each of the following numbers as a digit times a power of 10. [Ex: 4,000,000 = $4(10^6)$]

a) 50 _____

b) 0.5 _____

c) 80,000 _____

d) 800 _____

e) 0.09 _____

f) 9,000 _____

g) 600,000,000 _____

h) 0.006 _____

i) 30.000 _____

j) 30,000,000 _____

Multiplying and Dividing a Number By a Power of 10

In the last section you saw how trailing zeroes are carried along when you multiply by powers of 10. Adding trailing zeroes on is just like moving the decimal point.

To multiply a number by a power of 10, move the decimal point to the right the same number of places as the exponent.

$$49 \times 100 = 4,900$$
$$49.00 \times 100 = 4,900.$$

Since you are multiplying by 100 (10^2), move the decimal point 2 places to the right. Add zeroes when necessary .

$$325 \times 1,000 = 325,000$$
$$325.000 \times 1000 = 325,000$$

You can multiply by 1000 (10^3) by moving the decimal point 3 places to the right. Add zeroes when necessary.

Dividing by powers of 10 can be viewed in the same manner.

To divide a number by a power of 10, move the decimal point to the left the same number of places as the exponent.

$$\frac{6,000}{100} = 60.00 = 60$$

100 can be written as 10^2 , so you would move the decimal point 2 places to the left.

$$\frac{40,000}{10,000} = 4.0000 = 4$$

10,000 is the same as 10^4 , so you would move the decimal point 4 places to the left.

To make sense about which way to move the decimal point use the following tips:

1. By moving the decimal point to the right, you are making the number larger.
2. By moving the decimal point to the left, you are making the number smaller.

Example Set 4.

Complete the table using the technique of moving the decimal point to multiply and divide.

Number to Use	x 10	÷ 1,000	x 1,000
Ex. 308.5	3,085	0.3085	308,500
Ex. 5.004	50.04	0.005004	5,004
a) 730.2			
b) 2.105			
c) 45			
d) 0.08			

Scientific Notation

The population of California in 1990 was estimated at 29,600,000. Another way of writing large numbers like this, as well as small numbers, is to use a shorthand method called scientific notation. Thus, the population of California can be written as 2.96×10^7 . Scientific notation consists of two parts: a number with a value between 1 and 10 multiplied by a power of 10.

A.) 5,000,000,000 in scientific notation is 5×10^9 .

This is breaking the number into the digit times its place value.
(Count the O's.)

B.) 0.004 in scientific notation would be 4×10^{-3}

The 4 is in the thousandths place, so multiplying by 10^{-3} (1/1000) is the same as dividing by 10^3 (1,000).

C.) 8,200 in scientific notation is 8.2×10^3 .

Move the decimal point so that there is only one non-zero digit to the left of it. How many places did you move it? In what direction? 3 to the left (divided). Compensate for the division by multiplying the same number, 8.2×10^3 .

D.) 0.00029 in scientific notation is 2.9×10^{-4} .

Move the decimal point so that there is only one non-zero digit to the left. How many places did you move it? In what direction? 4 to the right (multiplied) Compensate for the multiplication by dividing the same number 2.9×10^{-4} .

E. 4.39×10^7 written the long way would be 43,900,000.

The single digit 4 is before the decimal point. Add enough zeros so that the 4 is in the 10^7 place. 43,900,000 (The 10^7 place means there are 7 digits between the 4 and the decimal point.)

Example Set 5.

Write the following numbers in scientific notation.

a) 70,000 _____

b) 300,000 _____

c) 800,000,000 _____

d) 9,000,000,000 _____

e) 0.008 _____

f) 0.00003 _____

g) 0.000009 _____

h) 0.0000002 _____

Example Set 6.

- a) In 1980, major airlines flew 5,400,000 flights. Write the number of flights in scientific notation.

- b) In 1986, ($\$1.25 \times 10^{10}$) was spent on state-run lotteries. Write this amount in long form.

- c) A pollen grain measures 0.0004 m in diameter. Write this measurement in scientific notation.

- d) The radius of the hydrogen atom is 10^{-8} cm. Write this in long form.

- e) To measure long distances in space, astronomers use a unit called a light-year. A light-year is approximately 5,880,000,000,000 miles long. Write this in scientific notation.

- f) Oprah Winfrey's salary as a TV talk show host was reportedly $\$3.5 \times 10^6$ for one year. Write this in long form.

- g) In 1990 the budget deficit reduction plan was to trim \$500 billion from the deficit. Write this figure in scientific notation.