Review Exercise Set 13

Exercise 1: Evaluate the given 2x2 determinant.

$$\begin{vmatrix} 3 & 8 \\ -2 & 5 \end{vmatrix}$$

Exercise 2: Evaluate the given 2x2 determinant.

$$\begin{vmatrix} 0.3 & -0.5 \\ 0.2 & 1 \end{vmatrix}$$

Exercise 3: Evaluate the given 3x3 determinant.

$$\begin{vmatrix} 2 & -3 & 4 \\ -1 & 1 & 2 \\ 5 & -2 & -3 \end{vmatrix}$$

Exercise 4: Use Cramer's rule to solve the given system of equations.

$$-x + y = 2$$
$$x - y = -4$$

Exercise 5: Use Cramer's rule to solve the given system of equations.

$$x - 2y + z = 0$$
$$-2x + y + 2z = 1$$
$$5x - 2y - 3z = 7$$

Review Exercise Set 13 Answer Key

Exercise 1: Evaluate the given 2x2 determinant.

$$\begin{vmatrix} 3 & 8 \\ -2 & 5 \end{vmatrix}$$

= (3 * 5) - (-2 * 8)
= 15 - (-16)
= 15 + 16
= 31

Exercise 2: Evaluate the given 2x2 determinant.

$$\begin{vmatrix} 0.3 & -0.5 \\ 0.2 & 1 \end{vmatrix}$$

= (0.3 * 1) - (0.2 * -0.5)
= 0.3 - (-0.1)
= 0.3 + 0.1
= 0.4

Exercise 3: Evaluate the given 3x3 determinant.

$$\begin{vmatrix} 2 & -3 & 4 \\ -1 & 1 & 2 \\ 5 & -2 & -3 \end{vmatrix}$$

Use row one to break the 3x3 determinant into a series 2x2 determinants

$$= (2)\begin{vmatrix} 1 & 2 \\ -2 & -3 \end{vmatrix} - (-3)\begin{vmatrix} -1 & 2 \\ 5 & -3 \end{vmatrix} + (4)\begin{vmatrix} -1 & 1 \\ 5 & -2 \end{vmatrix}$$
$$= (2)\begin{vmatrix} 1 & 2 \\ -2 & -3 \end{vmatrix} + (3)\begin{vmatrix} -1 & 2 \\ 5 & -3 \end{vmatrix} + (4)\begin{vmatrix} -1 & 1 \\ 5 & -2 \end{vmatrix}$$
$$= (2)\big[(1\times -3) - (-2\times 2)\big] + (3)\big[(-1\times -3) - (5\times 2)\big] + (4)\big[(-1\times -2) - (5\times 1)\big]$$
$$= (2)\big[-3+4\big] + (3)\big[3-10\big] + (4)\big[2-5\big]$$
$$= (2)(1) + (3)(-7) + (4)(-3)$$
$$= 2 - 21 - 12$$
$$= -31$$

Exercise 4: Use Cramer's rule to solve the given system of equations.

-x + y = 2
x - y = -4

Setup and evaluate the coefficient determinant D

$$\begin{aligned} D &= \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} \\ &= (-1 \times -1) - (1 \times 1) \\ &= 1 - 1 \\ &= 0 \end{aligned}$$

This system of equations is inconsistent and has no solution since D is zero, which would cause us to be dividing by zero when solving for x and y.

$$x = \frac{D_x}{D} = \frac{D_x}{0} = undefined$$
$$y = \frac{D_y}{D} = \frac{D_y}{0} = undefined$$

Exercise 5: Use Cramer's rule to solve the given system of equations.

x - 2y + z = 0
-2x + y + 2z = 1
5x - 2y - 3z = 7

Setup and evaluate the coefficient determinant D

$$\begin{aligned} D &= \begin{vmatrix} 1 & -2 & 1 \\ -2 & 1 & 2 \\ 5 & -2 & -3 \end{vmatrix} \\ &= (1)\begin{vmatrix} 1 & 2 \\ -2 & -3 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 2 \\ 5 & -3 \end{vmatrix} + (1)\begin{vmatrix} -2 & 1 \\ 5 & -2 \end{vmatrix} \\ &= (1)\left[(1 \times -3) - (-2 \times 2)\right] + (2)\left[(-2 \times -3) - (5 \times 2)\right] + (1)\left[(-2 \times -2) - (5 \times 1)\right] \\ &= (1)\left[-3 + 4\right] + (2)\left[6 - 10\right] + (1)\left[4 - 5\right] \\ &= (1)(1) + (2)(-4) + (1)(-1) \\ &= 1 - 8 - 1 \\ &= -8 \end{aligned}$$

Exercise 5 (Continued):

Setup and evaluate the determinant D_x

$$D_x = \begin{vmatrix} 0 & -2 & 1 \\ 1 & 1 & 2 \\ 7 & -2 & -3 \end{vmatrix}$$

$$D_x = (0)\begin{vmatrix} 1 & 2 \\ -2 & -3 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 2 \\ 7 & -3 \end{vmatrix} + (1)\begin{vmatrix} 1 & 1 \\ 7 & -2 \end{vmatrix}$$

$$= (0) + (2)\left[(1\times -3) - (7\times 2)\right] + (1)\left[(1\times -2) - (7\times 1)\right]$$

$$= (2)\left[-3-14\right] + (1)\left[-2-7\right]$$

$$= (2)(-17) + (1)(-9)$$

$$= -34 - 9$$

$$= -43$$

Setup and evaluate the determinant D_y

$$D_y = \begin{vmatrix} 1 & 0 & 1 \\ -2 & 1 & 2 \\ 5 & 7 & -3 \end{vmatrix}$$

$$D_y = (1)\begin{vmatrix} 1 & 2 \\ 7 & -3 \end{vmatrix} - (0)\begin{vmatrix} -2 & 2 \\ 5 & -3 \end{vmatrix} + (1)\begin{vmatrix} -2 & 1 \\ 5 & 7 \end{vmatrix}$$

$$= (1)\left[(1\times -3) - (7\times 2)\right] - (0) + (1)\left[(-2\times 7) - (5\times 1)\right]$$

$$= (1)(-3-14) + (1)(-14-5)$$

$$= (1)(-17) + (1)(-19)$$

$$= -17 - 19$$

$$= -36$$

Exercise 5 (Continued):

Setup and evaluate the determinant D_z

$$D_z = \begin{vmatrix} 1 & -2 & 0 \\ -2 & 1 & 1 \\ 5 & -2 & 7 \end{vmatrix}$$

$$D_z = (1)\begin{vmatrix} 1 & 1 \\ -2 & 7 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 1 \\ 5 & 7 \end{vmatrix} + (0)\begin{vmatrix} -2 & 1 \\ 5 & -2 \end{vmatrix}$$

$$= (1)\left[(1\times 7) - (-2\times 1)\right] + (2)\left[(-2\times 7) - (5\times 1)\right] + (0)$$

$$= (1)(7+2) + (2)(-14-5)$$

$$= (1)(9) + (2)(-19)$$

$$= 9 - 38$$

$$= -29$$

Determine the values of x, y, and z

$$x = \frac{D_x}{D} = \frac{-43}{-8} = 5\frac{3}{8}$$

$$y = \frac{D_y}{D} = \frac{-36}{-8} = 4\frac{1}{2}$$

$$z = \frac{D_z}{D} = \frac{-29}{-8} = 3\frac{5}{8}$$

The solution set is

(x, y, z) = $(5\frac{3}{8}, 4\frac{1}{2}, 3\frac{5}{8})$