Division of Polynomials

Objective A To divide a polynomial by a monomial

To divide a polynomial by a monomial, divide each term in the numerator by the denominator and write the sum of the quotients.

Simplify: $\dfrac{6x^3 - 3x^2 + 9x}{3x}$

$$= \frac{6x^3}{3x} - \frac{3x^2}{3x} + \frac{9x}{3x}$$
$$= 2x^2 - x + 3$$

Divide each term of the polynomial by the monomial.
Simplify each expression.

Simplify: $\dfrac{12x^2y - 6xy + 4x^2}{2xy}$

$$= \frac{12x^2y}{2xy} - \frac{6xy}{2xy} + \frac{4x^2}{2xy}$$
$$= 6x - 3 + \frac{2x}{y}$$

Objective B To divide polynomials

The procedure for dividing two polynomials is similar to the one for dividing whole numbers. The same equation used to check division of whole numbers is used to check polynomial division.

(Quotient x divisor) + remainder = dividend

Simplify: $(x^2 - 5x + 8) \div (x - 3)$

Step 1

$$x - 3 \overline{)\, x^2 - 5x + 8} \quad \Big| \; x$$

$$\underline{x^2 - 3x}$$

$$-2x + 8$$

Think: $x \overline{)x^2} = \frac{x^2}{x} = x$

Multiply: $x(x - 3) = x^2 - 3x$

Subtract: $(x^2 - 5x) - (x^2 - 3x) = -2x$

Step 2

$$x - 3 \overline{)\, x^2 - 5x + 8} \quad \Big| \; x - 2$$

$$\underline{x^2 - 3x}$$

$$-2x + 8$$

$$\underline{-2x + 6}$$

$$2$$

Think: $x \overline{)-2x} = \frac{-2x}{x} = -2$

Multiply: $-2(x - 3) = -2x + 6$

Subtract: $(2x + 8) - (2x + 6) = 2$

The remainder is 2.

Check: $(x-2)(x-3) + 2 = x^2 - 5x + 6 + 2 = x^2 - 5x + 8$

Therefore: $(x^2 - 5x + 8) \div (x - 3) = x - 2 + \frac{2}{x-3}$

If a term is missing from the dividend, a zero can be inserted for that term. This helps keep like terms in the same column.

Simplify: $(6x+26+2x^3)\div(2+x)$

$(2x^3+6x+26)\div(x+2)$

$$x+2\overline{)\,2x^3+0x^2+6x+26}\qquad 2x^2-4x+14$$

$$2x^3+4x^2$$

$$-4x^2+6x$$

$$-4x^2-8x$$

$$14x+26$$

$$14x+28$$

$$-2$$

Arrange the terms of each polynomial in descending order.

There is no x² term in $(2x^3+6x+26)\div(x+2)$. Insert a zero for the missing term.

Check: $(2x^2-4x+14)(x+2)+(-2)=(2x^3+6x+28)+(-2)=(2x^3+6x+26)$

Therefore: $(2x^3+6x+26)\div(x+2)=2x^2-4x+14-\dfrac{2}{x+2}$

Simplify: $(8x^2+4x^3+x-4)\div(2x+3)$

$(4x^3+8x^2+x-4)\div(2x+3)$

$$2x+3\overline{)\,4x^3+8x^2+x-4}\qquad 2x^2+x-1$$

$$4x^3+6x^2$$

$$2x^2+x$$

$$2x^2+3x$$

$$-2x-4$$

$$-2x-3$$

$$-1$$

Arrange the terms of each polynomial in descending order.

Check: $(2x^2 + x - 1)(2x + 3) + (-1) = (4x^3 + 8x^2 + x - 3) + (-1) = 4x^3 + 8x^2 + x - 4$

Therefore: $(4x^3 + 8x^2 + x - 4) \div (2x + 3) = 2x^2 + x - 1 - \dfrac{1}{2x + 3}$