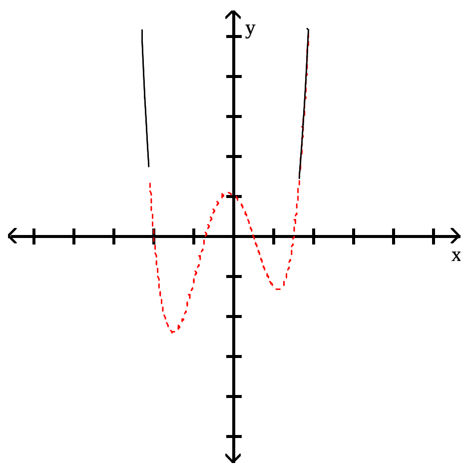
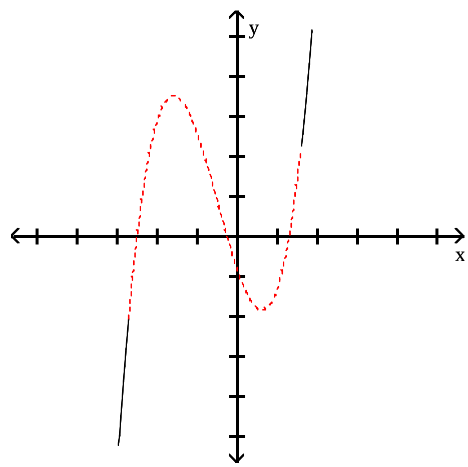
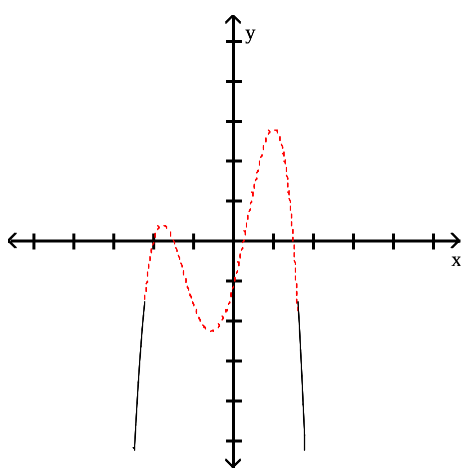
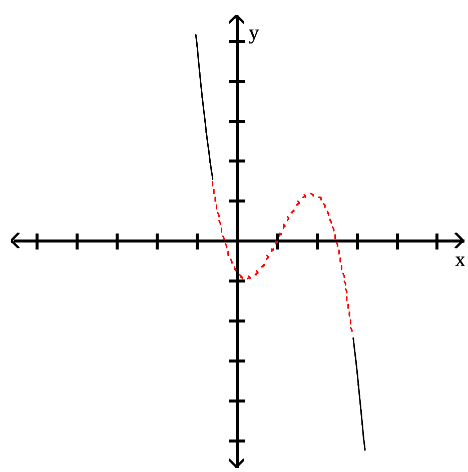
Steps To Graph Polynomial Functions

1. Make sure the function is arranged in the correct descending order of power.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \,, \quad \text{where the leading coefficient } a_n \neq 0$$

2. Determine the far–left and far–right behavior of the function.

<table>
<tr><th></th><th>n is even</th><th>n is odd</th></tr>
<tr><td>$a_n > 0$</td><td>up to the far–left
up to the far–right

</td><td>down to the far–left
up to the far–right

</td></tr>
<tr><td>$a_n < 0$</td><td>down to the far–left
down to the far–right

</td><td>up to the far–left
down to the far–right

</td></tr>
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3. Find the y–intercept.

a. To find $f(0)$, substitute zero for each x in the function.
b. Write your y–intercept in the form (0, __)
c. Plot this point.

4. Find any x–intercepts.

a. Set the function equal to zero.
b. Solve the resulting equation by factoring (or use the Rational Zeros Theorem to find the real zeros).
c. Write the x–intercepts in the form (__ , 0)
d. Plot these points.

NOTE: You will need the **factored form** found in this step to complete the rest of the steps.

$$f(x) = (x - a)(x + b)(x - c)\dots$$

5. Use the x–intercepts (zeros) to divide the x–axis into intervals and choose test a point in each interval. Determine the sign of all function values in that interval.

6. Use the multiplicity of each zero to determine where the graph crosses the x–axis.

For each factor $(x - c)^k$ of the function f:

- If k is **even**, the graph will intersect but **not** cross the x–axis at $(c, 0)$.
- If k is **odd**, the graph will cross the x–axis at $(c, 0)$.

7. If necessary, find additional points to draw an accurate graph.

- Substitute values for x into the factored form of the function, not the expanded form.
- Pay attention to signs when you are simplifying.

8. Sketch the graph.

Your graph should be a smooth, continuous curve that contains at most n x–intercepts and at most $(n - 1)$ **turning points**.