HOW ALGEBRA WORKS

CONCEPT	ILLUSTRATION
NOTATION FOR MULTIPLICATION	3×5 is used in arithmetic, but not usually in algebra because the times sign might be confused with an "x". Instead, we use the following to indicate multiplication:
Multiplication is indicated in several ways.	$3 \cdot 5$ 3(5) (3)5 (3)(5) 4(-5)(+3)
	$3 \cdot x$ 3(x) (3)x (3)(x) -7(-z)
	3x 3xy -2xyz -4(-x)(y)
TERMS AND FACTORS	
"Terms" are expressions that are added or subtracted.	3x + 4y contains two terms, 3x and 4y.
	$-5x^3yz$ is just one term.
"Factors" are multiplied together.	3x has two factors: 3 and x.
	(x + 4)(x - 8) has two factors: (x + 4) and (x - 8)
COEFFICIENTS	
In a term, the number that multiplies the letter or letters is called the "numerical coefficient" or just the "coefficient."	The coefficient of 3x is 3.
	NOTE: the coefficient of x is 1, because x = 1x. Also, -x = -1x.
EXPONENTS	
Exponents indicate how many factors are multiplied together.	x^3 means xxx
	NOTE: $x = x^1$
	$(x + 3)^2$ means (x + 3)(x + 3)

H2

ZERO AND NEGATIVE EXPONENTS

$x^0 = 1$ for *any* x except that 0^0 is undefined (not 0).

$4^0 = 1$ $(x + 6)^0 = 1$ if x is not -6

CAUTION: $(-3)^0 = 1$

BUT: $-3^0 = -1$

Also, for any x except 0,

$$x^{-n} = \frac{1}{x^n}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$3xy^{-2} = 3x\cdot\frac{1}{y^2} = \frac{3x}{y^2}$$

CAUTION: $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

BUT: $-3^{-2} = -\frac{1}{3^2} = -\frac{1}{9}$

MULTIPLYING POWERS OF THE SAME BASE

To multiply powers of the same base, multiply the coefficients but add the exponents.

$5x^3(2x^4) = 10x^7$

$(3x^{-3}y^3z^{-2})(-4x^5y^2z^{-4}) = -12x^2y^5z^{-6}$

$(5x^5)(5x^5) = 25x^{10}$

When exponents appear in a term, they are evaluated before multiplication, unless the multiplication is in parentheses.

$3x^2$ means 3xx; that is, x is squared FIRST and the RESULT is multiplied by 3.
But,
$(3x)^2$ means $(3x)(3x)$

CAUTION: $3 \cdot 4^2 = 3 \cdot 16 = 48$

BUT: $(3 \cdot 4)^2 = 12^2 = 144$

ALSO: $3^2 \cdot 2^4 \neq 6^6$

INSTEAD: $3^2 \cdot 2^4 = 9 \cdot 16 = 144$

CAUTION: $(-7)^2 = (-7)(-7) = 49$

***BUT*:** $-7^2 = -(7)(7) = -49$

RAISING A POWER TO A POWER

$(x^m)^n = x^{mn}$

$(x^3)^4$ means $(x^3)(x^3)(x^3)(x^3) = x^{12}$

$(y^3)^{-3} = y^{-9}$

RAISING A PRODUCT TO A POWER

$(ab)^n = a^n b^n$

$(xyz)^3 = x^3y^3z^3$

CAUTION: $(x + y)^2$ IS NOT EQUAL TO $x^2 + y^2$

USING TWO RULES AT ONCE

$(-3x^2y^{-3}z)^4 = (-3)^4(x^2)^4(y^{-3})^4z^4 = 81x^8y^{-12}z^4$

But if the first minus sign is outside the parentheses:

$-(3x^2y^{-3}z)^4 = -(3)^4(x^2)^4(y^{-3})^4z^4 = -81x^8y^{-12}z^4$

DIVIDING POWERS

$$\frac{x^m}{x^n} = x^{m-n}$$

$$\frac{6x^9}{8x^3} = \frac{3x^6}{4}$$

CAUTION: $\frac{x^9}{x^3} = x^6$ NOT x^3

ALSO, $\frac{x^3}{x^9} = \frac{1}{x^6} = x^{-6}$

SIMPLIFYING FRACTIONS WITH NEGATIVE EXPONENTS

To eliminate negative exponents in a fraction, multiply the numerator and denominator by corresponding positive powers.

$$\frac{3x^{-4}y^2z^{-1}}{y^{-3}x^3z^5} = \frac{3x^{-4}y^2z^{-1}}{y^{-3}x^3z^5} \cdot \frac{x^4zy^3}{x^4zy^3}$$

$$= \frac{3x^0y^5z^0}{y^0x^7z^6} = \frac{3y^5}{x^7z^6}$$

$$= \frac{x^{-2}+y}{z^{-1}} = \frac{x^{-2}+y}{z^{-1}} \cdot \frac{x^2z}{x^2z}$$

$$= \frac{x^0z + x^2yz}{x^2z^0} = \frac{z + x^2yz}{x^2}$$

LIKE TERMS

Terms that are equal except (possibly) for their coefficients are called like terms. Like terms--and ONLY like terms--may be combined by combining their coefficients. The result is another term LIKE those combined.

$3x + 4x = 7x$

$5x^3y - 6x^3y = -1x^3y = -x^3y$

$5xy - 7yx = -2xy$ (or $-2yx$)

NOTE: $x + x = 2x$

but $xx = x^2$

$3x + 3x = 6x$

but $(3x)(3x) = 9x^2$

NOTE: $5x^3 - 7x^3 = -2x^3$

but $5x^3(-7x^3) = -35x^6$

$6x^3(+5x^2) = 30x^5$

but $6x^3 + 5x^2$ cannot be simplified because the two terms are not like.

REMOVING PARENTHESES BY MULTIPLYING

In order to multiply two expressions together, multiply EACH TERM of one by EACH TERM of the other. After multiplying, combine like terms if possible.

$(a - b)(c + d - e)$
$= ac + ad - ae - bc - bd + be$

$(2x + 7)(3x - 8)$

$= 6x^2 - 16x + 21x - 56$

$= 6x^2 + 5x - 56$

FRACTIONS

It is okay to multiply a fraction by 1 without changing its value. Also, 1 may be written as anything over itself (except zero over zero).

$$\frac{2}{3} = \frac{2}{3}\cdot\frac{5}{5} = \frac{10}{15}$$

$$\frac{2x}{3y} = \frac{2x}{3y}\cdot\frac{5z}{5z} = \frac{10xz}{15yz}$$

EQUATIONS

It is okay to multiply both sides of an equation by ANYTHING that is not zero. Often, it helps to get rid of fractions by multiplying both sides of an equation by a common denominator.

One way to solve the equation

$$\frac{3x - 5}{4} = 7$$

is first to eliminate the fraction by multiplying both sides of the equation by 4.

COMMON DENOMINATORS

Any common denominator must contain all the denominators in the problem AS FACTORS (not terms). The LEAST common denominator (LCD) is the simplest common denominator.

EQUATION	LCD
$\frac{1}{2x} + \frac{1}{3x} = \frac{1}{6}$	6x
$\frac{1}{2(x - 5)} + \frac{1}{3(x + 6)} = \frac{1}{x}$	6x(x - 5)(x + 6)

LINES IN THE PLANE

In a plane, there is exactly one line between any two points. In mathematics, "line" means "straight line."

The SLOPE of a line is a number that indicates how "steep" the line is. A line going "uphill" from left to right will have a positive slope. A line going "downhill" from left to right will have a negative slope. Often, the letter "m" is used to stand for slope. A line through the two points (x_1, y_1) and (x_2, y_2) has slope calculated by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line through the points (3, -7) and (-4, -5) is:

$$\frac{-7 - (-5)}{3 - (-4)} = \frac{-7 + 5}{3 + 4} = \frac{-2}{7} = -\frac{2}{7}$$

The y's in the numerator must be subtracted in the same order as the x's in the denominator. Thus, the above slope could have been calculated like this:

$$\frac{-5 - (-7)}{-4 - 3} = \frac{-5 + 7}{-7} = \frac{2}{-7} = -\frac{2}{7}$$

H7

IMPORTANT!

* POINT-SLOPE FORM

A line with slope m which passes through the point (x_1, y_1) has equation:

$y - y_1 = m(x - x_1)$ (point slope form)

SLOPE-INTERCEPT FORM

A line with slope m and y-intercept b has equation:

$$y = mx + b \quad \text{(slope-intercept form)}$$

GENERAL FORM

The general form for the equation of a line is:

$$Ax + By = C$$

where A, B, and C are real numbers. Some books prefer the equivalent form:

$$Ax + By + C = 0$$

The line through (3, -5) with slope 2 has equation:

$$y - (-5) = 2(x - 3) \quad \text{(point-slope form)}$$

which can be simplified as follows:

$$y + 5 = 2x - 6$$

$$y = 2x - 6 - 5$$

$$y = 2x - 11 \text{ (slope-intercept form)}$$

Notice that the line goes through (0, -11) which is its y-intercept.

The equation

$$y = 2x - 11$$

can be changed from slope-intercept form to general form by putting the x's and y's on the left:

$-2x + y = -11$ (general form)

or

$-2x + y + 11 = 0$ (also general form)

HORIZONTAL LINES

Because all points on a horizontal line have the same y coordinate, the slope of every horizontal line is zero.

The line through (3, 7) and (5, 7) is horizontal. Its slope is:

$$\frac{7 - 7}{5 - 3} = \frac{0}{2} = 0$$

Because a horizontal line has slope 0, its equation can be written:

$y = 0x + b$

or just: $y = b$

where b is any real number and (0, b) is the y-intercept of the line.

The graph of $y = 7$ is a horizontal line through the point (0, 7).

The graph of $y = -3$ is a horizontal line through the point (0, -3).

The graph of $2y = 12$ is a horizontal line through the point (0, 6).

The graph of $y = 0$ is a horizontal line through the point (0, 0). THAT IS, THE EQUATION OF THE X-AXIS IS $y = 0$.

VERTICAL LINES

Because all points on a vertical line have the same x coordinate, the slope of every vertical line is undefined. That is, vertical lines have *no slope* or undefined slope, (which is *not* the same as a slope of zero).

The equation of a vertical line is

$x = h$

where h is any real number, and (h, 0) is the x-intercept of the line.

The line through (2, 6) and (2, 9) is vertical. Its slope would be:

$\frac{9 - 6}{2 - 2} = \frac{3}{0}$ which is undefined.

The graph of $x = 3$ is a vertical line through the point (3, 0).

The graph of $-5x = 10$ is a vertical line through the point (-2, 0).

The graph of $2x - 9 = 0$ is a vertical line through the point (4.5, 0).

The graph of $x = 0$ is a vertical line through the point (0, 0). THAT IS, THE EQUATION OF THE Y-AXIS IS $x = 0$.

H9

PARALLEL LINES

Two lines are parallel if and only if they have the same slope. (Of course, two vertical lines are parallel, even though they have no slope.)

The lines whose equations follow are all parallel:

$y = 3x - 9$	$y - 3x = 12$
$y = 3x$	$3x - y = 11$
$y = 3x + 7$	$2y = 6x + 105$

PERPENDICULAR LINES

Lines which meet at right angles (90°) are called perpendicular (abbreviated $\perp$). All horizontal lines are, of course, perpendicular to all vertical lines. Other lines are perpendicular if and only if their slopes are negative reciprocals of each other; that is, two nonvertical lines are perpendicular if and only if the slope of one is the reciprocal of the slope of the other, but with the *opposite* sign.

The line whose equation is:

$$y = \frac{2}{3}x + 11$$

is perpendicular to the lines whose equations are:

$$y = -\frac{3}{2}x + 6$$

$$y = -\frac{3}{2}x - \sqrt{17}$$

$$y = -\frac{3}{2}x$$

$$3x + 2y = 6$$

FINDING THE EQUATION OF A LINE

To figure the equation of a line, it is enough to know its slope and a point it contains.

Problem: in slope-intercept form, write the equation of the line through (3, -8) which is perpendicular to the line whose equation is: $3x + 2y = 10$.

Solution: the above equation can be written: $y = -\frac{3}{2}x + 5$ from which we see that its slope is $-\frac{3}{2}$. Therefore, the slope of the line we want must be $+\frac{2}{3}$. Because the line goes through (3, -8), its equation, in point-slope form, is:

$$y - (-8) = \frac{2}{3}(x - 3)$$

or: $y + 8 = \frac{2}{3}x - 2$

or: $y = \frac{2}{3}x - 2 - 8$

or: $y = \frac{2}{3}x - 10$ (slope-intercept form)

If required, we can write the equation in general form with integer coefficients (i.e., without fractions) by multiplying both sides of the last equation by 3 (the least common denominator) which yields:

$$3y = 2x - 30$$

and then rewriting the equation as any one of the following, which are all equivalent and all considered to be in general form with integer coefficients:

$$-2x + 3y = -30$$

or: $-2x + 3y + 30 = 0$

or: $2x - 3y - 30 = 0$

or: $2x - 3y = 30$

Equations and Graphs of Lines

Find an equation for the line with the following characteristics. Graph each line.

1. Through (0, -5) and parallel to $y = 3x$.
2. Through (0, -5) and parallel to $y = -3x$.
3. Through (0, 8) with slope $\frac{3}{4}$
4. Through (0, 8) with slope $-\frac{3}{4}$
5. Horizontal line through (3, 7).
6. Horizontal line through (6, -4).
7. Vertical line through (3, 8).
8. Vertical line through (-5, 6).
9. Through (0, 7) and parallel to $y = 2x - 15$.
10. Through (1, -6) and parallel to $y = 2x + 10$.
11. Through (0, π) and parallel to $2x - y = 14$.
12. Through (0, 0) and parallel to $y - 2x = -15$.
13. Through (-1, 5) and perpendicular to $y = -\frac{1}{3}x + 11$
14. Through (3, 4) and perpendicular to $y = 3x + \sqrt{11}$
15. Through (7, 12) and (-14, 3).
16. Through (6, -22) and (-3, -1).

Answers on page 45 and 63

Graphing Lines

Graph #1 and #2 on the same set of axes.

1. $y = 3x - 5$
2. $y = -3x - 5$

Graph #3 and #4 on the same set of axes.

3. $y = \frac{3}{4}x + 8$
4. $y = -\frac{3}{4}x + 8$

Graph # 5 and #6 on the same set of axes.

5. $y = 7$
6. $y = -4$

Graph # 7 and #8 on the same set of axes.

7. $x = 3$
8. $x = -5$

Graph #9, #10, #11, and #12 on the same set of axes. Note that they are parallel.

9. $y = 2x + 7$
10. $y = 2x - 8$
11. $y = 2x + \pi$
12. $y = 2x$

Graph #13 and #14 on the same set of axes. Note that they are perpendicular.

13. $y = 3x + 8$
14. $y = -\frac{1}{3}x + 5$

Graph #15 and #16 on the same set of axes. Note that they are perpendicular.

15. $y = \frac{3}{7}x + 9$
16. $y = -\frac{7}{3}x - 8$

Answers are on page 63

H 13

Name________________________________

Answers ↓	In slope-intercept form, write the equation of the line with the following characteristics ↓
1	Through (1, -3) with slope m = 5
2	Through (0, 32) and (100, 212)
3	Through (3, 9) and parallel to the line whose equation is $y = \frac{2}{3}x + \sqrt{11}$
4	Through (5, 4) perpendicular to the line whose equation is $y = \frac{5}{3}x - 30$
5	Through (3, -8) and perpendicular to the line whose equation is $3x + 2y = 10$
6	Horizontal line through (6, 3.5)
7	Vertical line through (6, 3.5)

H14

Name________________________________

Answers ↓	In slope-intercept form, write the equation of the line with the following characteristics ↓
1	Through (3, -1) with slope m = -4
2	Through (4, 3.6) and (-8, -5.4)
3	Through (-3, -7) and parallel to the line whose equation is $y = \frac{2}{3}x + \sqrt{11}$
4	Through (-5, 4) perpendicular to the line whose equation is $y = -\frac{5}{3}x - 30$
5	Through (6, -2) and perpendicular to the line whose equation is $2x - 3y = -24$
6	Horizontal line through $(9, 2\pi)$
7	Vertical line through $\left(\sqrt{7}, -8\right)$

H14A

Linear Regression

The *Least Squares Regression Line* is used to create a linear model to represent n data points of ordered pairs in the plane. In slope-intercept form, the equation of the line is $y = mx + b$, where

$$\text{slope} = m = \frac{n\sum(xy) - (\sum x)(\sum y)}{n\sum(x^2) - (\sum x)^2}$$

and

$$\text{intercept} = b = \frac{\sum y - m(\sum x)}{n}$$

Meaning of symbols.

Symbol	Meaning
n	The number of data points.
$\sum x$	Sum of all of the x values.
$\sum y$	Sum of all of the y values.
$\sum x^2$	Sum of all of the squares of all of the x values.
$\sum (xy)$	Sum of all of the products formed by multiplying each x value by the corresponding y value.
$\left(\sum x\right)^2$	The square of the sum of the x values.
$\left(\sum x\right)\left(\sum y\right)$	Multiply the sum of the x values by the sum of the y values.

H14B

Illustration. Determine the equation of the least-squares regression line for the following data.

x	3	6	9	12
y	4	11	15	20

x	y	x^2	xy
3	4	9	12
6	11	36	66
9	15	81	135
12	20	144	240
$\sum x = 30$	$\sum y = 50$	$\sum x^2 = 270$	$\sum (xy) = 453$
$n = 4$	$\left(\sum x\right)^2 = 900$		$\left(\sum x\right)\left(\sum y\right) = 1500$

$$m = \frac{n\sum(xy) - (\sum x)(\sum y)}{n\sum(x^2) - (\sum x)^2} = \frac{4 \cdot 453 - 1500}{4 \cdot 270 - 900} = \frac{312}{180} \approx 1.733$$

$$b = \frac{\sum y - m(\sum x)}{n} = \frac{50 - 1.733 \cdot 30}{4} \approx -0.498$$

The equation is: $y = mx + b$

$$y = 1.733x - 0.498$$

Now, on graph paper, plot the original 4 data points, then graph the above equation using the x and y intercepts.

H14 C

1. Suppose the following chart represents the number of cars sold as a function of their sticker prices.

Price in thousands	16	18	20	22	24	26	28
Number sold	125	102	83	76	81	44	22

Graph these data, where x represents the price and y represents the number sold.

Construct a demand function for these data using the least squares regression line. Use three-decimal-place accuracy.

x	y	x^2	xy
$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum (xy) =$
$n =$	$\left(\sum x\right)^2 =$	$\left(\sum x\right)\left(\sum y\right) =$	

$$m = \frac{n\sum(xy) - (\sum x)(\sum y)}{n\sum(x^2) - (\sum x)^2} = \_\_\_\_\_\_\_\_ \qquad b = \frac{\sum y - m(\sum x)}{n} = \_\_\_\_\_\_\_\_$$

2. Determine the Least Squares Regression Line that represents the two points (0, 32) and (100, 212). Do you recognize this equation?

H14D

Name________________________________

3. Suppose the following chart represents the population, y, of Squaresylvania (in thousands) x years after 1960.

x	0	10	20	30	40
y	100	115	120	135	150

(a) Graph these data on graph paper.

(b) Determine the equation of the least squares regression line, $y = mx + b$

(c) Graph the line on the same graph paper you used in part (a).

(d) Does the line look like a good model for the data?

(e) What does this model predict the population will be in 2010?

x	y	x^2	xy
$\sum x =$	$\sum y =$	$\sum x^2 =$	$\sum (xy) =$
$n =$ $\left(\sum x\right)^2 =$		$\left(\sum x\right)\left(\sum y\right) =$	

$$m = \frac{n\sum(xy) - (\sum x)(\sum y)}{n\sum(x^2) - (\sum x)^2} = \_\_\_\_\_\_\_\_ \qquad b = \frac{\sum y - m(\sum x)}{n} = \_\_\_\_\_\_\_\_$$

#14 E

Linear Regression Problems

1. As Earth's population continues to grow, the solid waste generated by the population grows with it. Governments must plan for disposal and recycling of ever growing amounts of solid waste. Planners can use data from the past to predict future waste generation and plan for enough facilities for disposing of and recycling the waste.

   Given the following data on the waste generated in Florida from 1990-1994, how can we construct a function to predict the waste that was generated in the years 1995-1999? The scatter plot is shown in Figure 1.85.

Year	Tons of Solid Waste Generated (in thousands)
1990	19,358
1991	19,484
1992	20,293
1993	21,499
1994	23,561

   a) Make a scatterplot of the data, letting x represent the number of years since 1990.
   b) Use a graphing calculator to fit linear, quadratic, cubic, and power functions to the data. By comparing the values of R^2, determine the function that best fits the data.
   c) Graph the function of best fit with the scatterplot of the data.
   d) With each function found in part (b), predict the average tons of waste in 2000 and 2005, and determine which function gives the most realistic predictions.

2. The numbers of insured commercial banks y (in thousands) in the United States for the years 1987 to 1996 are shown in the table. (Source: Federal Deposit Insurance Corporation).

Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
y	13.70	13.12	12.71	12.34	11.92	11.46	10.96	10.45	9.94	9.53

   Make a scatterplot of the data, letting x represent the number of years since 1987.
   a) Use a graphing calculator to fit linear, quadratic, cubic, and power functions to the data. By comparing the values of R^2, determine the function that best fits the data.
   b) Graph the function of best fit with the scatterplot of the data.
   c) With each function found in part (b), predict the average number of insured commercial banks in 2000 and 2005, and determine which function gives the most realistic predictions.
   e) Plot the actual data *and* the model you selected on the same graph. How closely does the model represent the data?

#14 F

H14G

H14 K

H14 I

H143

H14K

Name________________________ T11310Rev 205201200

__________ 1. If (x, y) = (5, -4) then $xy^3 - xy = ?$

2. Simplify: $(-5)^2 =$ ______ $-(5^2) =$ ______ $-5^2 =$ ______

__________ 3. Remove parentheses: $-3x^2(-3x^3 + 4x^2 - 6x + 9)$

__________ 4. Solve: $4x - 7 = 5x - 9$

__________ 5. Solve: $-4(7 + 3x) = 11 + 3(-5x - 4)$

__________ 6. Solve: $z - \frac{2}{3} = \frac{1}{2} + \frac{5}{6}z$

__________ 7. Simplify: $\frac{x - 3}{x^2 - 5x + 6}$

__________ 8. Divide: $(-6x^3 + 19x^2 - 27x - 20) \div (3x - 5)$

(x, y) = __________ 9. Solve the system: $5x - 4y = -43$
$-2x + 3y = 27$

Problems 10 and 11, factor completely over the integers:

__________ 10. $6x^2y - 9xy^2 - 3xy$

__________ 11. $12x^2 - 11x - 15$

Problems 12 and 13: solve by factoring.

__________ 12. $x^2 - 5x - 6 = 0$

__________ 13. $3x^2 = 7x + 40$

ANSWERS ON pages ~~55-58~~ H17-H

H16

Name____________________________ T11310Rev 205201200

Problems 14, 15, 16: solve using the quadratic formula.

____________ 14. $x^2 - x - 1 = 0$

____________ 15. $3x^2 = 7x + 40$

____________ 16. $x^2 - 5x - 6 = 0$

__________ 17. LITE cheese has $\frac{3}{5}$ less calories than their regular cheese. How many slices of LITE cheese contain the calories of 10 slices of regular cheese?

________________ 18. What should the selling price of a house be in order to have $59,850 left after paying a real estate agent 5% of the selling price?

Problems 19 and 20, graph; show the intercepts, if any.

19. $-5x = 10 - 2y$

20. $-y = 3x - 10$

Quadratic formula

IF $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

① $(x,y) = (5,-4)$ $\quad xy^3 - xy$

$= (5)(-4)^3 - (5)(-4)$

$= 5(-64) - (-20)$

$= -320 + 20$

$\boxed{= -300}$ Ans

---

② $(-5)^2 = (-5)(-5) = \boxed{25}$

$-(5^2) = -(5 \cdot 5) = \boxed{-25}$

-5^2 means $-5 \cdot 5 = \boxed{-25}$ NOT 25!

---

③ $-3x^2(-3x^3 + 4x^2 - 6x + 9)$

$= \boxed{9x^5 - 12x^4 + 18x^3 - 27x^2}$ Ans

---

④ $4x - 7 = 5x - 9$

$4x - 5x = -9 + 7$

$-x = -2$

$\boxed{x = 2}$

OR

$4x - 7 = 5x - 9$

$-7 + 9 = 5x - 4x$

$\boxed{2 = x}$

SAME

---

⑤ $-4(7 + 3x) = 11 + 3(-5x - 4)$

$-28 - 12x = 11 + (-15x) - 12$

$-28 - 12x = -1 - 15x$

$-12x + 15x = -1 + 28$

$3x = 27$

$\frac{3x}{3} = \frac{27}{3}$

$\boxed{x = 9}$

(6) $z - \frac{2}{3} = \frac{1}{2} + \frac{5}{6}z$

common denominator = 6

$6\left[z - \frac{2}{3}\right] = 6\left[\frac{1}{2} + \frac{5}{6}z\right]$

$6z - \frac{12}{3} = \frac{6}{2} + \frac{30}{6}z$

$6z - 4 = 3 + 5z$

$6z - 5z = 3 + 4$

$\boxed{z = 7}$

OR

$z - \frac{2}{3} = \frac{1}{2} + \frac{5}{6}z$

$z - \frac{5}{6}z = \frac{1}{2} + \frac{2}{3}$

$\frac{6}{6}z - \frac{5}{6}z = \frac{3}{6} + \frac{4}{6}$

$\frac{1}{6}z = \frac{7}{6}$ ⟸ (now multiply by 6)

$\boxed{z = 7}$

---

(7) $\frac{x-3}{x^2-5x+6} = \frac{x-3}{(x-3)(x-2)} = \boxed{\frac{1}{x-2}}$

---

(8)

$$-2x^2 + 3x - 4 + \frac{-40}{3x-5}$$

$3x-5 \overline{)\,-6x^3 + 19x^2 - 27x - 20}$

$\pm 6x^3 \mp 10x^2$

$9x^2 - 27x$

$-9x^2 \pm 15x$

$-12x - 20$

$\pm 12x \mp 20$

$\boxed{-40}$

---

(9) $2[5x - 4y = -43] \longrightarrow 10x - 8y = -86$

$5[-2x + 3y = 27] \quad -10x + 15y = 135$

$7y = 49$

$\boxed{y = 7}$

$3[5x - 4y = -43] \longrightarrow 15x - 12y = -129$

$4[-2x + 3y = 27] \quad -8x + 12y = 108$

$7x = -21$

$\boxed{x = -3}$

$\boxed{(-3, 7)}$

H19

1. Ensure that the divisor and dividend are written in descending powers of the variable. (You many need to insert a "missing" power with a coefficient of zero.)	$3x-5\overline{)-6x^3+28x^2-42x-20}$
2. Copy the first term of the dividend.	$3x-5\overline{)-6x^3+28x^2-42x-20}$ Copy: $-6x^3$
3. Figure the first term in the quotient as follows: the first term of the divisor, $3x$, times the first term of the quotient must equal the first term in the dividend, $-6x^3$.	$-2x^2$ $3x-5\overline{)-6x^3+28x^2-42x-20}$ $-6x^3$
4. Multiply the $-2x^2$ by the -5 as well.	$-2x^2$ $3x-5\overline{)-6x^3+28x^2-42x-20}$ $\underline{-6x^3+10x^2}$
5. Subtract by *changing the signs* and combining. "Bring down" the next term in the quotient.	$-2x^2$ $3x-5\overline{)-6x^3+28x^2-42x-20}$ $+\quad -$ (change the signs) $\underline{-6x^3+10x^2}$ $+18x^2-42x$
6. Repeat the process of copying, figuring the next term of the answer, multiplying, and subtracting until you're done. If there is a remainder, add it to the quotient as the numerator of a fraction whose denominator is the divisor.	$-2x^2+6x-4+\frac{-40}{3x-5}$ $3x-5\overline{)-6x^3+28x^2-42x-20}$ $+\quad -$ $\underline{-6x^3+10x^2}$ $+18x^2-42x$ $-\quad +$ $\underline{+18x^2-30x}$ $-12x-20$ $+\quad -$ $\underline{-12x+20}$ -40

H20

(10) $3xy(2x - 3y - 1)$

(11) $12x^2 - 11x - 15 = (4x + 3)(3x - 5)$ ANS

CHECK: $12x^2 - 20x + 9x - 15 = 12x^2 - 11x - 15$ ✓

(12) $x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x-6 = 0$ or $x+1 = 0$

$x = 6$ or $x = -1$ ANS

(13) $3x^2 = 7x + 40$

$3x^2 - 7x - 40 = 0$

$(3x + 8)(x - 5) = 0$

$3x + 8 = 0$ or $x - 5 = 0$

$x = 5$

$3x = -8$

$x = -8/3$ or $x = 5$

(14) $x^2 - x - 1 = 0$

$ax^2 + bx + c = 0$

$a = 1 \quad b = -1 \quad c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

(15) $3x^2 = 7x + 40$

$3x^2 - 7x - 40 = 0$

$ax^2 + bx + c = 0$

$a = 3$

$b = -7$

$c = -40$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-40)}}{2(3)}$$

$$\frac{7 \pm \sqrt{49 + 480}}{6} = \frac{7 \pm \sqrt{529}}{6}$$

$$= \frac{7 \pm 23}{6} = \frac{7+23}{6}, \frac{7-23}{6} = \frac{30}{6}, \frac{-16}{6} = 5, -8/3$$

#21

TEST 1 Review PAGE 4

(16)

16) $x^2-5x-6=0$

$ax^2+bx+c=0$

$a=1 \quad b=-5 \quad c=-6$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-6)}}{2(1)}=\frac{5\pm\sqrt{25+24}}{2}$$

$$=\frac{5\pm\sqrt{49}}{2}=\frac{5\pm7}{2}=\frac{12}{2},\frac{-2}{2}=\boxed{6,-1}$$

---

17) $$\frac{1}{1-\frac{3}{5}}\cdot\frac{10}{1}=\frac{1}{\frac{2}{5}}\cdot\frac{10}{1}=\frac{5}{2}\cdot\frac{10}{1}=\frac{50}{2}=\boxed{25}$$

#17

#17

---

18) $x-.05x=59850$

#18

$.95x=59850$

$$x=\frac{59850}{.95}=63000$$

---

19) $-5x=10-2y$

#19

x	y
0	
	0

if $x=0$

$0=10-2y$

$2y=10$

$y=5$

$(0,5)$ is y-int

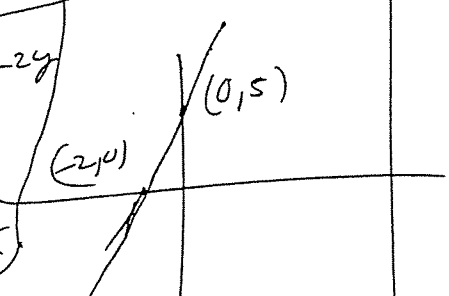
x	y
0	5
-2	0

$y=0$, $-5x=10-2y$

$-5x=10$

$x=-2$

$(-2,0)$ is x-int



20) $-y=3x-10$

x	y
0	
	0

if $x=0$, $-y=-10$
$y=10$

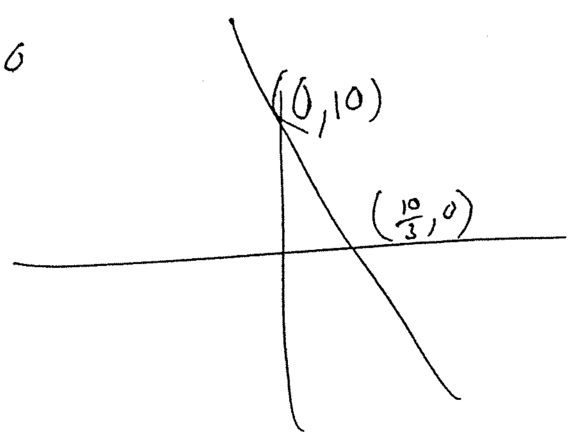
if $y=0$

$0=3x-10$

$10=3x$

$\frac{10}{3}=x$

x	y
0	10
$\frac{10}{3}$	0



(58)

Suppose we are asked to factor the trinomial $60x^2 - 115x - 120$. In order for this process to work as described below, we must first factor out the common monomial factor:

$60x^2 - 115x - 120 = 5(12x^2 - 23x - 24)$ (See the next page for a method of determining the GCD of any two integers.)

Now we use the quadratic formula to solve the related quadratic equation, $12x^2 - 23x - 24 = 0$.

Recall the quadratic formula. If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Thus, if $12x^2 - 23x - 24 = 0$, then

$$x = \frac{-(-23) \pm \sqrt{(-23)^2 - 4(12)(-24)}}{2(12)}$$

$$= \frac{23 \pm \sqrt{529 + 1152}}{24} = \frac{23 \pm \sqrt{1681}}{24}$$

$$= \frac{23 \pm 41}{24} = \frac{23+41}{24} \text{ or } \frac{23-41}{24} = \frac{64}{24} \text{ or } = \frac{-18}{24} = \frac{8}{3} \text{ or } = \frac{-3}{4}$$

The critical observation is that if we know that either $x = a$ or $x = b$, then it must be true that $(x - a)(x - b) = 0$. In this case, we must have: $\left(x - \frac{8}{3}\right)\left(x - \frac{-3}{4}\right) = 0$

But the original equation was $12x^2 - 23x - 24 = 0$. In order to get the leading coefficient to be 12, we multiply both sides by 12. Of course, $12(0) = 0$. We get:

$$12\left(x - \frac{8}{3}\right)\left(x - \frac{-3}{4}\right) = 0$$

We note that $12 = (3)(4)$. In order to cancel the denominators, we replace 12 with 3 times 4, rearrange the factors, and multiply as follows:

$$3\left(x - \frac{8}{3}\right)4\left(x - \frac{-3}{4}\right) = 0$$

$$(3x - 8)(4x - (-3)) = 0$$

$(3x - 8)(4x + 3) = 0$. We now check by multiplying out the parentheses:

$12x^2 + 9x - 32x - 24 = 12x^2 - 23x - 24$ ☺

How can we determine the GCD of any two integers without guessing? One method is the Euclidean Algorithm. Let's use the Algorithm to find the GCD of 12 and 68.

The process consists of dividing 12 into 68, and then dividing each remainder into the previous divisor until the remainder is zero.

```
12)68(5
   60
    8)12(1
       8
       4)8(2
         8
         0
```

When we get a remainder of 0, we are done. The last divisor, 4, is the GCD of 12 and 68.

What if the GCD is 1? No problem. Let's find the GCD of 98 and 199.

```
98)199(2
   196
     3)98(32
       96
        2)3(1
          2
          1)2(2
            2
            0
```

The last divisor, 1, is the GCD of 98 and 199.

Okay, let's finish with the GCD of 1666 and 4216.

```
1666)4216(2
     3332
      884)1666(1
           884
           782)884(1
               782
               102)782(7
                   714
                    68)102(1
                    68
                     34)68(2
                        68
                         0
```

N.B. Suppose we are to find GCD (A, B, C).
Let x = GCD(A, B). Then
GCD (A, B, C) = GCD (x, C).

The last divisor, 34, is the GCD of 1666 and 4216.

#24

Exponents and Logarithms

Exponents. Recall that 10^2 = 100, 10^3 = 1000, 10^4 = 10,000, etc. It turns out that there is a useful definition that extends exponents to any rational number: $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$ Here, we assume that *m* and *n* are positive integers, the exponent is in lowest terms, and $n \geq 2$. (Discuss: What would it mean if $n = 1$?) Also, if *n* is even and *x* is negative, then imaginary numbers are involved. The definitions $x^0 = 1$ and $x^{-n} = \frac{1}{x^n}$ allow exponents to be any rational numbers. In higher math, we further extend exponents to cover all numbers, giving meaning to things like $x^{\pi}, x^{\sqrt{2}}$ and even x^i where $i = \sqrt{-1}$. Using the definition of fractional exponents, see if you can simplify the following: $27^{-\frac{2}{3}}$.

Fill in the following chart and use it to sketch the graph of the functions: $y = 10^x$, $y = 2^x$, $y = 3^x$, and $y = \left(\frac{1}{2}\right)^x = \frac{1^x}{2^x} = \frac{1}{2^x} = 2^{-x}$

x	0	1	2	3	4	5	-1	-2	-3	-4
$y = 10^x$										
$y = 2^x$										
$y = 3^x$										
$y = 2^{-x}$										

Mathematicians have defined irrational exponents in such a way that numbers like 2^{π} "fill in" the holes making all of these graphs smooth and continuous.

Logarithms. Notice from your graph of $y = 10^x$ that, **if N is any positive number, then N is equal to 10 raised to *some* power.** ***That power is called the logarithm of N to the base 10.*** In symbols, if $10^y = x$ then $y = \log_{10} x$, which for convenience is abbreviated simply "$\log x$" and is called the "common log" of *x*. The graph of $y = \log x$ can be obtained by making a table of values for the equivalent function, $10^y = x$. (In the table below, instead of replacing *x* with values and calculating *y*, it is easier to replace *y* with numbers on the top row and calculate *x* on the bottom row.) Fill in the table and use it to verify that the graph of $y = \log x$ is the mirror image of the graph of $y = 10^x$, where the "mirror" is the line whose equation is $y = x$.

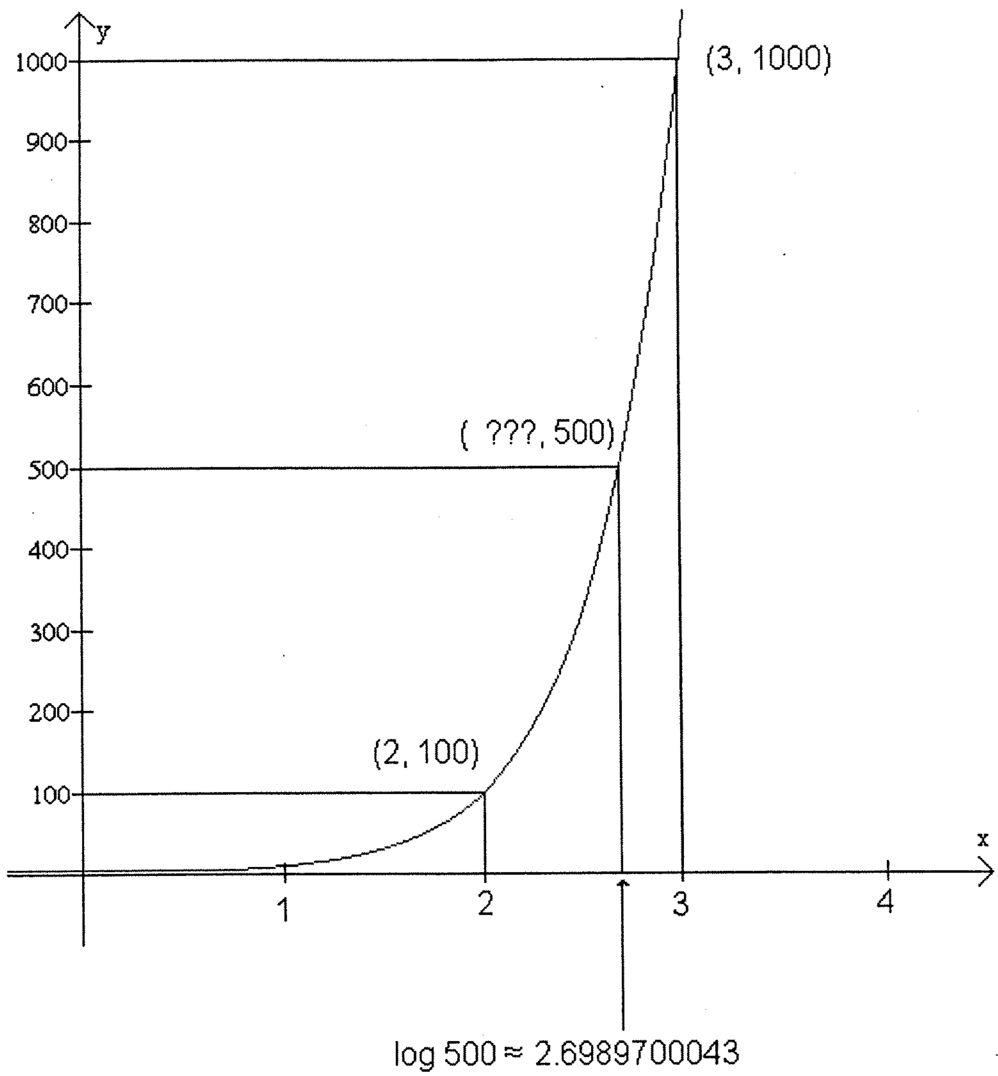
$y = \log x$	0	1	2	3	4	5	-1	-2	-3	-4
$x = 10^y$										

Although any positive number (other than 1) can be the base of a logarithm, we use two bases quite frequently. They are base 10 and base *e*, where *e* (which is about 2.718) is an important irrational number (like π), used frequently in mathematics. We also use an abbreviation for base *e* logarithms, namely: $\log_e x = \ln x$. The "ln" stands for "log natural," and ln *x* is called the "natural log" of *x*. Because $e^{\ln x} = x$ and $\ln e^x = x$, the functions $y = e^x$ and $y = \ln x$ are examples of inverse functions, which we will study shortly.

H25

Below is a graph of $y = 10^x$

Verify that the graph contains the points (2, 100) and (3, 1000).



Because every positive number is equal to the number 10 raised to *some* power, there must be a particular value of x such that $10^x = 500$. This number is called the *logarithm of 500 to the base 10.* Its approximate value is 2.6989700043; the exact value is an irrational number (like π) whose decimal representation has infinitely many digits. Fundamentally, **A LOGARITHM IS AN EXPONENT.**

Thus, $10^{2.6989700043} \approx 500$ (approximately), whereas $10^{\log 500} = 500$ (exactly).

If you have a scientific calculator, experiment with it to determine if you obtain the logarithm of 500 by entering "500 LOG" or "LOG 500."

H26

Concept	Examples
This is the fundamental property of logarithms: $\log x = y$ means $10^y = x$	$\log x = 5$ means $10^5 = x$ Or: $x = 100{,}000$
Base 10 logs are called "common logs." The other base frequently used in mathematics, and available on a scientific calculator, is base e, where $e \approx 2.71828$. Base e logarithms are called "natural logs" and are abbreviated "ln" which stands for "log natural." Hence, $\ln x = y$ means $e^y = x$	$\ln x = 5$ means $e^5 = x$ See if you can use your scientific calculator to determine that $e^5 \approx 148.41315910257660$
Any positive number other than 1 can be the base for a logarithm. The log of x to the base B is written: $\log_B x$ As before, $\log_B x = y$ means $B^y = x$	The log of 30 to the base 5 is written $\log_5 30$. The equation, $\log_5 30 = y$ means $5^y = 30$
If we need to know the logarithm of a number to a base other than 10 or e, we use the "change of base" formula which calculates the required log using base 10 or base e. $\log_B x = \dfrac{\log x}{\log B} = \dfrac{\ln x}{\ln B}$	Use a scientific calculator to verify that $\log_5 30 \approx 2.113282752559$ whether you use the change of base formula with base 10 or base e. $\log_5 30 = \dfrac{\log 30}{\log 5} = \dfrac{\ln 30}{\ln 5}$

H27

Recall the fundamental property of logarithms: $\log_B x = y$ means $B^y = x$	Use the fundamental property of logs to solve the following. $\log_2 x = 4$ $\log_3 27 = y$ $\log_B 64 = 3$	
Recall that $10^{\log 500} = 500$. In general, $10^{\log x} = x$ $e^{\ln x} = x$ $B^{\log_B x} = x$	Evaluate: $10^{\log 47} =$ $e^{\ln 5.4} =$ $B^{\log_B 789} =$	
The log of the base raised to a power is just the power: $\log 10^x = x$ $\ln e^x = x$ $\log_B B^x = x$	Suppose that you are asked to evaluate (or simplify) $\log 10^x$ Begin by writing $\log 10^x = y$ which, by the fundamental property, means that $10^y = 10^x$ Do you see why $y = x$?	
Recall that. $\log_B x = y$ means $B^y = x$ Use this fact to write exponential equations that are equivalent to the logarithmic equations to the right.	Logarithmic equation: $\log_2 16 = 4$ $\log 10000 = 5$ $\ln 25 \approx 3.218875824868$	Equivalent exponential equation:
Write logarithmic equations equivalent to the exponential equations to the right.	$3^5 = 243$ $e^y = x$ $10^y = x$	

#28

Properties of logarithms

Here, we will focus on common logs (base 10), but these results hold for any base.

We will prove the following rules for manipulating logarithms:

1. $\log(uv) = \log u + \log v$
2. $\log\left(\frac{u}{v}\right) = \log u - \log v$
3. $\log u^p = p \log u$
4. $\log_B x = \frac{\log x}{\log B} = \frac{\ln x}{\ln B}$
5. $\log\frac{1}{u} = -\log u$

We will need to use these two properties of logarithms:

$10^{\log x} = x$ (Here, x must be positive. Why?)

$\log 10^x = x$ (Here, x can be positive, negative, or zero. Why?)

Rule 1: If u and v be any positive numbers, $\log(uv) = \log u + \log v$	
Let $U = \log u$ and $V = \log v$	
Then	Provide reasons for the steps on the left.
$u = 10^U$ and $v = 10^V$	______________________
$\log(uv) = \log(10^U \cdot 10^V)$	______________________
$\log(uv) = \log 10^{U+V}$	______________________
$\log(uv) = U + V$	______________________
$\log(uv) = \log u + \log v$	______________________
This proves rule 1.	
See if you can prove Rule 2 by mimicking the proof for Rule 1.	Rule 2. $\log\left(\frac{u}{v}\right) = \log u - \log v$

#29

Rule 3. If u is any positive number and p is any real number (positive, negative, or zero), then: $\log u^p = p \log u$	
Let $U = \log u$ $u = 10^U$ $\log u^p = \log(10^U)^p$ $= \log 10^{pU}$ $= pU$ $= p \log u$ This proves rule 3.	Provide reasons for the steps on the left. ____________________________ ____________________________ ____________________________ ____________________________ ____________________________
Rule 4. $\log_B x = \frac{\log x}{\log B} = \frac{\ln x}{\ln B}$ Proof. Let $\log_B x = z$ Then $B^z = x$ (Why?) Finish the proof by taking the base 10 (or base e) log of both sides, using Rule 3, and solving for z.	
Rule 5. $\log \frac{1}{u} = -\log u$	See if you can prove this on your own. ☺

H30

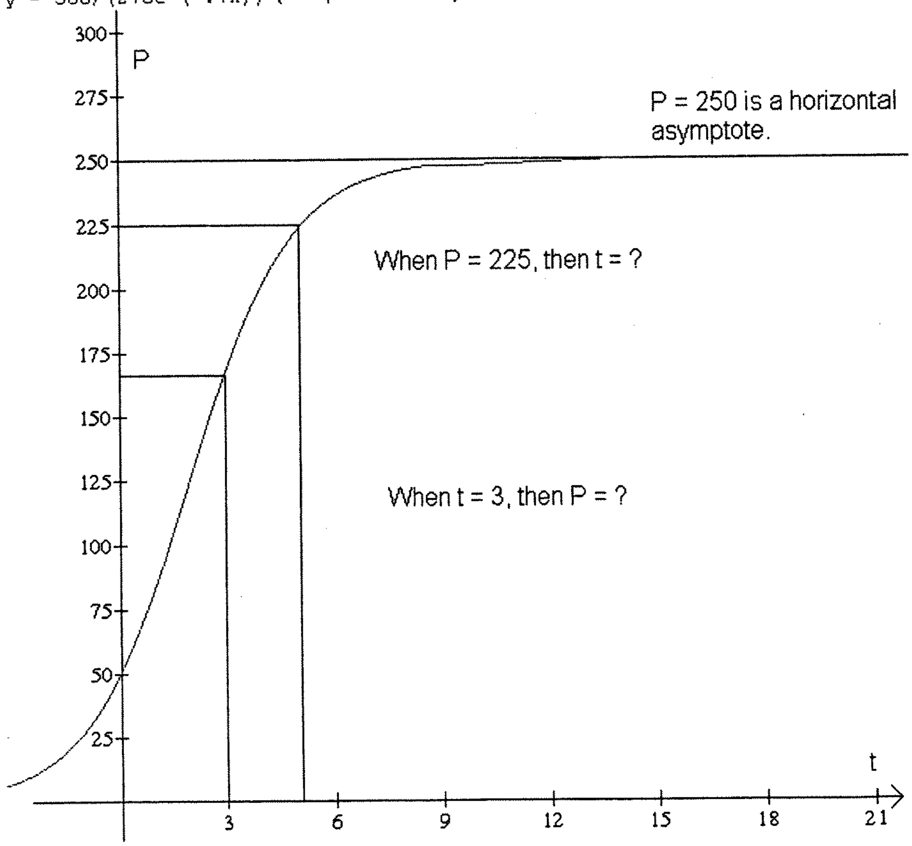
The following properties are stated for base 10, but they hold for any base. Note that the argument for the logarithmic function must be positive. Expressions such as log (-5) are not real numbers; they are discussed in advanced mathematics.

$\log(uv) = \log u + \log v$	$\log(500 \cdot 300) = \log 500 + \log 300$
$\log\left(\frac{u}{v}\right) = \log u - \log v$	$\log\left(\frac{27}{350}\right) = \log 27 - \log 350$
$\log u^p = p \log u$ Before the age of calculators, this fact was used to find roots of numbers. For example, how do you think logarithms could be used in calculating the cube root of 23? Note that $\sqrt[3]{23} = 23^{\frac{1}{3}}$ Now what?	$\log 27^5 = 5 \log 27$ $\log 73^{-\frac{2}{3}} = -\frac{2}{3}\log 73$ $\log x^{-3} = -3 \log x$ $\log \sqrt{x} = \log x^{\frac{1}{2}} = \frac{1}{2}\log x$
Recall: $\log 10^x = x$ $10^{\log x} = x$ $\ln e^x = x$ $e^{\ln x} = x$ $\log_B B^x = x$ $B^{\log_B x} = x$	Simplify: $\log 10^{2.1}$ $10^{\log 34}$ $\ln e^{7.3}$ $e^{\ln \pi}$ $\log_{12} 12^8$ $5^{\log_5 97}$
In order to simplify the expressions above, it is important that the base of the logarithm and the base of the exponent are the same. In particular: $\log e^x \neq x$ $\ln 10^x \neq x$ $10^{\ln x} \neq x$ $e^{\log x} \neq x$	To calculate $\log e^5$, first determine the fifth power of e, then take the log (base 10) of that number. Thus: $\log e^5 \approx \log 148.413159102576603$ $\approx 2.171472409516259$ Note: Mixing base 10 and base e is more of a curiosity than something one regularly encounters, even in higher math.
Write as a single logarithm: $\log(x+5) + \log(x-5) - 2\log(x+7)$	Solve: $\log(x+3) + \log(x-2) - \log 14 = 0$

H31

Logs are extremely useful in solving equations like $5^x = 30$. Just take the log of both sides to any convenient base, such as 10 or *e*, and use the power property: $\log u^p = p \log u$ In the illustration on the right, verify that you get the same answer if you use the natural log (base *e*) rather than the common log (base 10).	Illustration: Solve: $5^x = 30$ $\log 5^x = \log 30$ $x \log 5 = \log 30$ $x = \dfrac{\log 30}{\log 5} \approx \dfrac{1.4771212}{0.6989700} \approx 2.1132827$
A conservation organization releases 50 animals of an endangered species into a game preserve. The organization estimates that the preserve can support about 250 animals and that the growth of the herd will be modeled by the logistics curve: $P(t) = \dfrac{500}{2 + 8e^{-0.7t}}$ (See graph below.)	According to the mathematical model to the left, what will be the approximate population of the herd in 3 years? When does the model predict that the population will reach 225?

y = 500/(2+8e^(-.7x)) (Winplot formula)



Factoring aid for using the "ac" method of factoring $ax^2 + bx + c$

Here are some illustrations of how one might use the table on the following pages.

1. Find $|ac|$ in the left column.

2. If, $ac > 0$, find $|b|$ in the third column; if $ac < 0$, find $|b|$ in fourth column.

3. Illustration. Factor: $6x^2 - 103x - 35$.

Since 6(-35) = -210, locate the number 210 in the left column.

$\|ac\|$	Positive factors of $\|ac\|$	**If $ac > 0$,** Find $\|b\|$ = Sum of positive factors in this column.	**If $ac < 0$,** Find $\|b\|$ = Difference of positive factors in this column.
210	1, 210	211	209
	2, 105	107	103
	3, 70	73	67
	5, 42	47	37
	6, 35	41	29
	7, 30	37	23
	10, 21	31	11
	14, 15	29	1

Noting that $ac < 0$, look at the fourth column, find $|b| = 103$, and then replace $-103x$ with $2x - 105x$, (or with $-105x + 2x$) and proceed to factor by grouping.

H33

For each integer from 1-225, determine all the pairs of factors as well as the sum and difference of each pair.

- The number of lines provided for each number in the left column will tell you how many pairs of factors you must find.
- Note that prime numbers and the number 1 are associated with only one pair of factors.
- Note that if $|ac|$ is the product of exactly two primes, then there are exactly two pairs of factors associated with $|ac|$.
- If you are factoring the difference of two squares, then the rightmost column will tell you that $|b| = 0$, which also appears in rows corresponding to factoring a perfect square trinomial.
- Note that if n is a positive integer and n is not prime, then n must have a prime factor no larger than the $\sqrt{n}$. Thus, if no prime less than or equal to $\sqrt{n}$ is a divisor of n, then n must be prime.
- If you determine the pairs of factors of n beginning with 1, 2, 3,..., you may stop when you get to $\sqrt{n}$.
- Note that each pair of factors for n consists of the same set of prime factors. For example, 210 = (2)(3)(5)(7), and these four primes constitute every pair of factors of 210. Illustration.

Pairs of factors for 210	This is an example of "the Fundamental Theorem of Arithmetic": a positive integer greater than 1 has exactly one set of prime factors. Note:
1, 210	(1)210 = (2)(3)(5)(7)
2, 105	(2)105 = (2)(3)(5)(7)
3, 70	(3)70 = (3)(2)(5)(7)
5, 42	(5)42 = (5)(2)(3)(7)
6, 35	(6)35 = (2)(3)(5)(7)
7, 30	(7)30 = (7)(2)(3)(5)
10, 21	(10)(21) = (2)(5)(3)(7)
14, 15	(14)(15) = (2)(7)(3)(5)

H34

$\lvert ac \rvert$	Positive factors of $\lvert ac \rvert$	If $ac > 0$, Find $\lvert b \rvert$ = Sum of positive factors in this column.	**If $ac < 0$,** Find $\lvert b \rvert$ = Difference of positive factors in this column.
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			

H35

18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			
31			
32			
33			
34			
35			

H36

36			
37			
38			
39			
40			
41			
42			
43			
44			
45			
46			
47			
48			
49			
50			
51			

H37

52			
53			
54			
55			
56			
57			
58			
59			
60			
61			
62			
63			

H38

64			
65			
66			
67			
68			
69			
70			
71			
72			
73			
74			
75			
76			
77			
78			

H39

79			
80			
81			
82			
83			
84			
85			
86			
87			
88			
89			

90			
91			
92			
93			
94			
95			
96			
97			
98			
99			
100			
101			

102			
103			
104			
105			
106			
107			
108			
109			
110			
111			
112			
113			

114			
115			
116			
117			
118			
119			
120			
121			
122			
123			
124			
125			

126			
127			
128			
129			
130			
131			
132			
133			
134			
135			
136			
137			

138			
139			
140			
141			
142			
143			
144			
145			
146			
147			
148			
149			

150			
151			
152			
153			
154			
155			
156			
157			
158			
159			
160			

161			
162			
163			
164			
165			
166			
167			
168			
169			
170			
171			
172			
173			

174			
175			
176			
177			
178			
179			
180			
181			
182			
183			
184			
185			

186			
187			
188			
189			
190			
191			
192			
193			
194			
195			

H49

196			
197			
198			
199			
200			
201			
202			
203			
204			
205			
206			
207			

H50

208			
209			
210	1, 210	211	209
	2, 105	107	103
	3, 70	73	67
	5, 42	47	37
	6, 35	41	29
	7, 30	37	23
	10, 21	31	11
	14, 15	29	1
211			
212			
213			
214			
215			
216			
217			
218			

H51

219			
220			
221			
222			
223			
224			
225			